What Does It Really Mean to Be College and Work Ready?

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The Mathematics Required of First Year Community College Students

NCEE
National Center On EDUCATION And The Economy®

What Does It Really Mean to Be College and Work Ready?

The Mathematics Required of First Year Community College Students

A Report from the National Center on Education and the Economy

May 2013

NCEE
National Center On
EDUCATION
And The Economy ®

The National Center on Education and the Economy was created in 1988 to analyze the implications of changes in the international economy for American education, formulate an agenda for American education based on that analysis and seek wherever possible to accomplish that agenda through policy change and development of the resources educators would need to carry it out.

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PREFACE

In the fall of 2009, the National Center on Education and the Economy initiated a series of research programs designed to support our high school reform program, *Excellence for All*, based on our more than 20 years of research on the school reform programs of the countries with the most successful education programs worldwide.

The design of *Excellence for All* entails the use of some of the world's best instructional programs, with high quality curriculum and high quality matching examinations. To make this program work as designed, we had to be sure that the performance standards we identified as "passing" on the lower division exams we had selected in English and mathematics were sufficiently challenging that students passing these examinations were likely to be successful in the first year of a typical community college program.

We asked the association of community colleges what that standard might be. They did not know. There was no shortage of opinion about what it might be, much of it based on asking panels of community college faculty for the answer. But this method of determining education standards is notoriously faulty, because educators, job foremen and others presumably in a position to know typically answer in terms of what they would like students and workers to know and be able to do, not in terms of what the program of study or the work *actually* requires. We quickly discovered that no one had done in-depth research on what was needed to be successful in our community colleges.

So we set in motion two empirical studies, one focused on English and the other on mathematics requirements. Each of these studies was guided by a panel of leading experts in that subject matter area, including key figures from the community colleges themselves, as well as leading subject matter experts and researchers. Both studies were conducted under the aegis and watchful eye of the *Excellence for All* Technical Advisory Committee, whose members include many of the nation's leading psychometricians, cognitive scientists, curriculum experts and testing experts. I am deeply indebted to both the subject matter committees and the Technical Advisory Committee for the time and careful attention they have given to these studies over the two-and-a-half years it has taken to conduct them. Special appreciation goes to the Mathematics Panel co-chairs, Phil Daro and Sol Garfunkel, for their leadership, thoughtfulness and creativity in steering the Panel through the challenging tasks we set before them.

Most of the work, as is usually the case, was done by the staff. Betsy Brown Ruzzi, NCEE's Vice-President for Programs, produced the original research design and has continued to be deeply involved in the work. Jackie Kraemer, Senior Policy Analyst, conducted the research. Jennifer Craw, Production Designer, assembled and aggregated all the data coding and developed the data displays. David R. Mandel, Director of Research and Policy Analysis, oversaw the whole process and played a key role in drafting the reports.

This entire effort also enjoyed the support and encouragement of the Bill & Melinda Gates Foundation as part of their College Ready Education strategy.

The nation is, at long last, engaged in serious discussion of what it means to be College and Career Ready. We believe that this research program will make an important contribution to that debate by cutting through strongly expressed opinions on the matter that turn out to be just plain wrong in the light of our findings, findings that may surprise many observers.

But these findings will not surprise all. As the facts presented in these reports came to light in the course of our research, I shared them with people very close to the institutions we were researching. Few of them were surprised. Most told me that the emerging picture corresponded closely to what they saw every day in the field. They had long ago concluded that the debate about standards was unhinged from the realities in our community colleges.

We offer these research reports in the hope that our findings will make an important contribution to the larger debate about what it means to be college and career ready and what our schools should be doing to provide a curriculum that will help all students be ready for college and careers when they graduate.

Some may charge that our findings constitute an argument to lower high school leaving standards. That would be a gross misreading of our findings. A large fraction of high school graduates cannot now do the work required of them in the first year of the typical community college program. Our first priority should be to reduce that fraction greatly by teaching all high school students what they will need to succeed in college. Instead, we do not teach what they need, while demanding of them what they don't need: furthermore the mathematics that we do teach and that they do need, we teach ineffectively. Perhaps that is the place to begin our deliberations.

Marc Tucker, President
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I. SUMMARY

THERE IS GROWING CONSENSUS in the United States that high school students should leave high school ready to succeed in both college and work. But there is less agreement as to what that actually means. Several states have adopted the position that, in the field of mathematics, it means that all high school students should be required to master Algebra II (Advanced Algebra) as a condition of getting a diploma. But is it true that students who do not take Algebra II will find themselves unable to succeed at either college or work? What is required to be successful in our nation's colleges and workplaces?

This report answers these questions with new empirical data. And its findings may surprise the reader.

We focused our research on the requirements of community colleges, because, by doing so, we can provide a very concrete image of what it means to be "college and career ready." First, a very large fraction of our high school graduates go on to attend these institutions, which some have described as the workhorse of our postsecondary education system. Second, our community colleges provide the bulk of the serious vocational and technical education taking place in the United States below the baccalaureate level, for everyone from auto mechanics and nurses to emergency medical technicians and police officers. If a student cannot successfully complete a community college two-year certificate or degree program leading directly to such a job, that student will have a very hard time supporting a family above the poverty line. Third, our community colleges provide the first two years of a four-year college program leading to a baccalaureate degree for another large fraction of their students. So it is reasonable to say that if a student leaves high school unable to succeed in the initial credit-bearing courses in their local community college, that student is ready neither for work nor college. And we know that, in fact, a large proportion of our high school graduates are indeed unable to succeed in their first year in community college. So now we can be much more precise in our question. How much and what kind of mathematics does a student have to know and be able to deploy to be successful in their initial credit-bearing community college courses?

This research began by looking at a sample of seven community colleges in seven states. We focused on nine of the most popular and diverse programs in these colleges - Accounting, Automotive Technology, Biotech/Electrical Technology, Business, Computer Programming, Criminal Justice, Early Childhood Education, Information Technology, Nursing - and the General Track. We collected data on the mathematics that are actually taught in the initial credit bearing courses in those programs, and in the initial mathematics courses these programs require students to take. We did this by analyzing the textbooks and exams and other work assignments used in these courses.

Only one program in one college required entering students to have mastered the content of Algebra II before enrolling in that program. Algebra II is an integral element in the sequence of mathematics courses that are required of students who will go on to take calculus and to use calculus in their work, but that is true of only about five percent of the working population.[1]

1 See Anthony P. Carnevale, Nicole Smith and Michelle Melton, *STEM* (Washington, DC: Center on Education and the Workforce, Georgetown University, 2011).

Indeed, community college first year programs of study typically assume that students have not mastered Algebra I. The most advanced mathematics content used in the vast majority of the first-year college programs we analyzed can reasonably be characterized as the mathematics associated with Algebra 1.25, that is some, but not all, of the topics usually associated with Algebra I, plus a few other topics, mostly related to geometry or statistics. Most of the mathematics that is required of students before beginning these college courses and the mathematics that most enables students to be successful in college courses is not high school mathematics, but middle school mathematics, especially arithmetic, ratio, proportion, expressions and simple equations.

Considering the importance of middle school mathematics content, it should be of real concern that a large proportion of our high school graduates do not have a sound command of this fundamental aspect of mathematics. We also found that many students, to be successful in our community colleges, need to be competent in some areas of mathematics that are rarely taught in our elementary or secondary schools, such as schematics, geometric visualization and complex applications of measurement.

In sum, a substantial part of the high school mathematics we teach is mathematics that most students do not need, some of what is needed in the first year of community college is not taught in our schools, and the mathematics that is most needed by our community college students is actually elementary and middle school mathematics that is not learned well enough by many to enable them to succeed in community college. A significant body of research on teacher knowledge, including the work of Liping Ma, Jim Stigler and Deborah Ball, makes it clear that one reason for this is because the instruction in arithmetic, ratio, proportion, expressions and simple equations that our teachers have received in school and in college falls far short of what it needs to be for them to have a sound conceptual grasp of the mathematics they are asked to teach.[2]

We conclude the following:

1. Many community college career programs demand little or no use of mathematics. To the extent that they do use mathematics, the mathematics needed by first year students in these courses is almost exclusively middle school mathematics. But the failure rates in our community colleges suggest that many of them do not know that math very well. A very high priority should be given to the improvement of the teaching of proportional relationships including percent, graphical representations, functions, and expressions and equations in our schools, including their application to concrete practical problems.

2. Whatever students did to pass mathematics courses in middle school, it does not appear to require learning the concepts in any durable way. While they may have been taught the appropriate procedures for solving certain standard problems, the high rates of non-completion by the significant percentages of students who arrive at college with the most modest command of mathematics suggests that there are significant weaknesses in teaching the concepts on which these procedures are based. This is a very serious problem that needs to be addressed in the first instance by the way mathematics is taught to prospective teachers of elementary and middle school mathematics in the arts and sciences departments of our universities and the way mathematics education is taught in our schools of education.

3. It makes no sense to rush through the middle school mathematics curriculum in order to get to advanced algebra as rapidly as possible. Given the strong evidence that mastery of middle school mathematics plays a very important

[2] A recent addition to this body of knowledge that examines what developmental mathematics students in community college actually understand about mathematics (James W. Stigler, Karen B. Givvin, and Belinda J. Thompson, "What Community College Developmental Mathematics Students Understand about Mathematics," *MathAMATYC Educator,* v1 n3 (May 2010): p 4-16 reinforces prior findings that the dominance of attention to procedure in K-12 mathematics education accompanied by lack of focus on conceptual understanding contributes significantly to students struggling with middle school mathematics and algebra in college.

role in college and career success,[3] strong consideration should be given to spending more time, not less, on the mastery of middle school mathematics, and requiring students to master Algebra I no later than the end of their sophomore year in high school, rather than by the end of middle school. This recommendation should be read in combination with the preceding one. Spending more time on middle school mathematics is in fact a recommendation to spend more time making sure that students understand the concepts on which all subsequent mathematics is based. It does little good to push for teaching more advanced topics at lower grade levels if the students' grasp of the underlying concepts is so weak that they cannot do the mathematics. Once students understand the basic concepts thoroughly, they should be able to learn whatever mathematics they need for the path they subsequently want to pursue more quickly and easily than they can now.

4. Mastery of Algebra II is widely thought to be a prerequisite for success in college and careers. Our research shows that that is not so. The most demanding mathematics courses typically required of community college students are those required by the mathematics department, not the career major, but the content of the first year mathematics courses offered by the community colleges' mathematics department is typically the content usually associated with Algebra I, some Algebra II and a few topics in geometry. It cannot be the case that one must know Algebra II in order to study Algebra I or Algebra II. Based on our data, one cannot make the case that high school graduates must be proficient in Algebra II to be ready for college and careers.

   The high school mathematics curriculum is now centered on the teaching of a sequence of courses leading to calculus that includes Geometry, Algebra II, Pre-Calculus and Calculus. However, fewer than five percent of American workers and an even smaller percentage of community college students will ever need to master the courses in this sequence during college or in the workplace.[4] There is a clear case for including the topics in this sequence in the high school curriculum as an option for students who plan to go into careers demanding mastery of these subjects, but they should not be required courses in our high schools. To require these courses in high school is to deny to many students the opportunity to graduate high school because they have not mastered a sequence of mathematics courses they will never need. In the face of these findings, the policy of requiring a passing score on an Algebra II exam for high school graduation simply cannot be justified.

5. Our research shows that many of the most popular community college programs leading to well-paying careers require mathematics that is not now included in the mainstream high school mathematics program, including mathematical modeling (how to frame a real-world problem in mathematical terms), statistics and probability. Our research also shows that success in many community college programs demands mastery of certain topics in mathematics that are rarely, if ever, taught in American elementary and secondary schools, including complex applications of measurement, geometric visualization and schematic diagrams. American high schools should consider abandoning the requirement that all high school students study a program of mathematics leading to calculus and instead offer that mathematics program as one among a number of options available for high school students in mathematics, with other options available (e.g., statistics, data analysis and applied geometry) that include the mathematics needed by workers in other clusters of occupations. By doing so high schools will almost certainly expand opportunity to many

[3] See Stigler, Givven and Thompson and the findings of the mathematics gaining the greatest attention in the community college majors that comprise the heart of this study.

[4] See Anthony P. Carnevale, Nicole Smith and Michelle Melton, *STEM* (Washington, DC: Center on Education and the Workforce, Georgetown University, 2011) for their forecast of the growing percentage of STEM jobs, not all of which will require calculus.

students who now find success in college closed off by a one-size-fits-all sequence of mathematics topics that actually fits the requirements only for a very narrow range of occupations.

6. The research we did revealed a major gap in the alignment between the mathematics courses taught in the mathematics departments in our community colleges and the mathematics actually needed to be successful in the applied programs students are taking. In some of the cases we observed, the departments offering the applied programs apparently felt compelled to create their own mathematics courses rather than require a course in the mathematics department. In a great many cases, the mathematics department course had little or nothing to do with the actual mathematics required to be successful in the applied programs the students were enrolled in. It may well be that many community college students are denied a certificate or diploma because they have failed in a mathematics course focused on mathematics topics that are irrelevant to the work these students plan to do or the courses they need to take to learn how to do that work. That strikes us as unfair. Because this is true and because we also noted that students in the applied programs often need mathematics that was never offered in high school or in college, we think the community colleges need to review their mathematics requirements in the light of what has been learned about what students need to know about mathematics to be successful in the careers they have chosen.

7. Like the standard high school mathematics sequence, the placement tests that community colleges use to determine whether students will be allowed to register for credit-bearing courses or be directed instead to take remedial courses in mathematics are based on the assumption that all students should be expected to be proficient in the sequence of courses leading to calculus, in particular that they should be expected to be proficient in the content typically associated with Algebra I, Algebra II and Geometry. But our research, as we have noted, shows that students do not need to be proficient in most of the topics typically associated with Algebra II and much of Geometry to be successful in most programs offered by the community colleges. This is a very serious issue. It is clear that many students are being denied entry to credit-bearing courses at our community colleges who are in fact prepared to do the mathematics that will be required of them in their applied programs.

   A very large proportion of students who enroll in remedial programs fail to get a degree or certificate, whether or not they complete their remedial programs. It follows that a large fraction of students applying to our community colleges are needlessly running up debt taking remedial courses they do not need to take to be successful in the applied programs of their choice, and are in the process being denied access to the programs that could make all the difference between rewarding careers and lives on the one hand and lives of poverty and frustration on the other. The research showing that many students who fail their placement tests in mathematics but then go on to be successful in community college, makes the point.[5]

8. While the textbooks in the introductory program courses were often impressive in their demand for mathematical thinking, the tests were a different story. Judging by the tests community college teachers administer to their students in the introductory program courses in their career majors, their courses are typically pitched to the lower set of expectations described by Bloom's hierarchy—memorization of facts and mastery of procedures—and not to the kinds of analytical skills, writing ability, ability to synthesize material to put together solutions to problems the student has not seen before and other complex skills that employers are now demanding. Community colleges need to review their course and program objectives

[5] Clive Belfield and Peter M. Crosta, *Predicting Success in College: The Importance of Placement Tests and High School Transcripts*, (New York: Community College Research Center, 2012) and Judith Scott Clayton, *Do High Stakes Placement Exams Predict College Success?* (New York: Community College Research Center, 2012).

in the light of current employer demands to make sure that they are helping their students develop the kinds of skills that will make their graduates employable.

9. What is tested by community college instructors typically falls far short of what is contained in the texts those teachers assign to their students. Judging by what is tested by community college teachers, they do not typically appear to be requiring students to apply mathematics or even to think mathematically even when the text they have chosen for the courses uses math to explain relevant phenomena or presents mathematical skills as an important element in the skills required to do the work. It is not clear whether this is because the teachers do not think that that material is in fact needed for success in the workplace or because, although they do think it is needed, they do not think their students capable of learning the material. This, too, is a very important issue. If it is the case that many community college teachers are teaching less material than they think is actually needed or teaching material at a lower level than they think the work actually demands, because they do not believe their students can absorb the material they actually need to absorb, then our community colleges are shortchanging our students and this problem needs to be addressed.

Decisions on setting standards for college readiness have typically been based on surveys of college professors and on predictive studies associating high school course-taking with college grades. But neither tells us what we want to know. College professors, when asked what they think high school students need to know to do college-level work, typically tell researchers what they would like students to know and be able to do, rather than what is actually required. Predictive studies based on the names of courses students take (a student needs to get a grade of at least X in a course named Y to have high probability of getting a grade of Z in their initial credit bearing college courses) assume that the content of high school courses with the same course names is the same and that the name of the course accurately describes its content, but there is no evidence that that is true. That is why it is so important to do empirical research of the sort we have done to find out what is actually taught at the community college year-one level and to base one's conception of what it means to be college-and-career ready on such research.

Many observers will be surprised by the low expectations that community colleges have for their students and will therefore respond to our findings by saying that, by making the actual requirements for community college the definition of "college and work ready," we would be setting the standards for high school too low. We have in fact suggested that community colleges should review their offerings in the light of what is known about the current demands of work as described by employers, but we should bear in mind that a large fraction of high school graduates cannot now meet the low standards we describe in this report. We think it would be wise and certainly fairer to do what is necessary to enable all of our students to meet the current low standards than to raise the standards before we have enabled both our schools and community colleges to do a better job of preparing our students for a more rigorous community college curriculum. When they are better prepared, it is very likely that community college faculty would themselves raise the standards for community college achievement. The first challenge, and it is enormous, will be to make sure our high school graduates are prepared for success in the community college programs we have today.

II. BACKGROUND

THIS REPORT SUMMARIZES one of several research initiatives the National Center on Education and the Economy (NCEE) is undertaking as part of its *Excellence for All* initiative. *Excellence for All* aims to bring the world's best aligned instructional systems to American high schools. Such systems are characterized by: 1) courses that constitute a coherent core curriculum, typically consisting, at a minimum, of courses in one's native language, mathematics, the sciences, history and the arts, each of which is framed by a detailed syllabus; 2) instructional materials custom tailored to support each curriculum; 3) teacher professional development that trains teachers to teach the curriculum and organize instruction so that the broad range of students likely to encounter it will succeed; 4) high quality examinations (typically dominated by essays and other forms of constructed response tasks) that are designed to assess the extent to which students have command of the material described in the syllabus and can apply it to unfamiliar problems; and 5) professional scoring of the examinations.

For more than 20 years, NCEE has been benchmarking the national education systems that perform at the top of the international league tables and get large percentages of their students ready to be successful in college. One of the key findings emerging from this work is that one of the most important reasons why these nations have overtaken the U.S. in student achievement is that they have powerful, coherent and aligned instructional systems. This finding is less of a secret today than it once was as other researchers have come to similar conclusions. As NCEE has shared these results with states and localities it has found good numbers interested in putting such systems into place and in the fall of 2011 they began doing so.

NCEE is now working in four states to pilot two different systems, ACT's *QualityCore* program and the University of Cambridge's International General Certificate of Secondary Education (IGCSE) program, in 39 high schools. One of the key provisions of the pilot is that students who succeed in these "lower division" (9^{th} and 10^{th} grade) programs and do well on their examinations will have the option to enter open admissions postsecondary colleges and immediately begin taking credit-bearing courses without any need to take remedial ones. They will also be able to stay in high school and complete an "upper division" program, such as the International Baccalaureate, the Advanced Placement Diploma Program or Cambridge A-Levels all of which provide solid preparation for highly selective colleges and universities and in many cases offer college credit as well.

It is very important to the success of this plan that the passing scores on the English and mathematics examinations accurately reflect what it takes to be successful in initial credit-bearing courses at open admissions colleges and universities. This study was designed to support this effort by identifying what knowledge and skills in mathematics are actually required in such courses.

With these factors providing a rationale for conducting this work, our immediate goals were as follows:

- To describe the mathematics in the initial required mathematics courses and the initial courses in a variety of program areas in open admissions colleges and universities;
- To describe what the prerequisite mathematics is for students to be successful in these courses given the mathematics found in these courses; and
- To inform the process of setting qualification scores on the aligned instructional system's examinations to better ensure that students will succeed in credit-bearing courses at open admissions colleges and universities, given current demands.

III. METHODOLOGY

IN ORDER TO ADDRESS THESE ISSUES, NCEE collected course materials (syllabi, required texts, graded mid-term and final exams and, in some cases, graded assignments that count toward the final grade) from first-year courses at seven randomly selected community colleges in seven states that were interested in this work. These colleges are in Arizona, Connecticut, Kentucky, Mississippi, New Hampshire, New Mexico and New York. They serve a mix of rural, urban and suburban populations and their enrollments range from 3,000 to 30,000.

The standard way of determining what students need to know and be able to do to succeed in college is to put this question to highly regarded faculty. What usually results is their aspirational notions of what is desirable rather than what is currently needed. To make sure that our findings reflected actual needs rather than aspirations, NCEE gathered and analyzed actual evidence of the sort noted above to determine reading, writing and mathematical literacy knowledge and skills needed to succeed. This was done in nine highly popular and diverse program areas (Accounting, Automotive Technology, Biotech/Electrical Technology, Business, Computer Programming, Criminal Justice, Early Childhood Education, Information Technology, and Nursing), as well as the initial mathematics and English courses required by each of these programs.[6] We did not analyze any certificate programs, only programs that led to an AA, AAS or AS degree or allowed students to transfer to a four-year institution to continue studying for a BA or BS degree. In each case, the first required courses for the general track were covered in the set of courses we analyzed.

In order to analyze the mathematics in these courses (i.e., the first required mathematics courses as well as the introductory or 101 courses for each program), a panel of mathematics experts drawn from community colleges as well as four-year institutions and other venues was assembled (see Biographical Sketches in Appendix A).

To review and analyze the evidence, a methodology was developed based on the content in the *Common Core State Standards for Mathematics* (CCSSM) and the math competencies from the OECD's Programme for International Student Assessment (PISA). The CCSSM are organized by grade level for kindergarten to eighth grade and then by the major mathematics subject areas typically covered in high school. For this project, three bands of standards, K-5, 6-8 and high school, were created. The panel conducted the analysis at the "cluster level" of the CCSSM, as it was felt this level of detail was sufficient without being too fine-grained (see List of Content Codes in Appendix B). The coding charts provide the findings by "domains" which are groupings of clusters, except in a few cases where there was an interest in analyzing the findings in more detail. The PISA competencies include six dimensions: Symbols and Formalism; Reasoning and Argumentation; Solving Problems Mathematically; Modeling; Communication; and Representation. Each are spelled out in this framework that follows.

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[6] About one-third of community college students who graduate choose to major in the liberal arts and sciences, general studies and/or humanities, a figure that has remained steady over the last decade. The next most popular majors are in the health professions and related clinical sciences, which encompass about 21% of all associate degrees granted. Business is another popular major, drawing 15.7% of community college students, followed by engineering at 6.5%. Security and protective services and computer and information services round out the most popular majors with 4.4% and 3.8% of students choosing these fields, respectively. While health fields have experienced an increase in graduates between the 1999-00 and 2009-10 school years (from 15.3% to 20.9%), engineering has dropped from 10.5% of graduates to just 6.5%. Most other fields have remained fairly stable. (NCES, *Condition of Education, 2012*, (2012). Washington, DC)

PISA Mathematics Expert Group Item-Difficulty Coding Framework[7]

SYMBOLS AND FORMALISM

Variable-Definition	Understanding, *manipulating*, and *making use* of symbolic expressions within a mathematical context (including arithmetic expressions and operations), governed by mathematical *conventions and rules*; understanding and *utilising constructs* based on definitions, rules and *formal systems*.
Level 0	No mathematical rules or symbolic expressions need to be activated beyond fundamental arithmetic calculations, operating with small or easily tractable numbers.
Level 1	Make direct use of a simple functional relationship, either implicit or explicit (for example, familiar linear relationships); use formal mathematical symbols (for example, by direct substitution or sustained arithmetic calculations involving fractions and decimals) or activate and directly use a formal mathematical definition, convention or symbolic concept.
Level 2	Explicit use and manipulation of symbols (for example, by algebraically rearranging a formula); activate and use mathematical rules, definitions, conventions, procedures or formulae using a combination of multiple relationships or symbolic concepts.
Level 3	Multi-step application of formal mathematical procedures; working flexibly with functional or involved algebraic relationships; using both mathematical technique and knowledge to produce results.

REASONING AND ARGUMENTATION

Variable-Definition	Logically rooted thought processes that explore and link problem elements so as to *make inferences* from them, or to *check a justification that is given* or *provide a justification* of statements.
Level 0	Make direct inferences from the instructions given.
Level 1	Reflect to join information to make inferences, (for example to link separate components present in the problem, or to use direct reasoning within one aspect of the problem).
Level 2	Analyze information (for example to connect several variables) to follow or create a multi-step argument; reason from linked information sources.
Level 3	Synthesize and evaluate, use or create chains of reasoning to justify inferences or to make generalizations, drawing on and combining multiple elements of information in a sustained and directed way.

[7] Mogens Allen Niss, Ross Turner, John Dossey and Werner Blum, "Using Mathematical Competencies to Predict Item Difficulty in PISA: A MEG Study" in Research on PISA: Research Outcomes of the PISA Research Conference of 2009, edited by Manfred Prenzel, Mareike Kobarg, Katrin Schöps and Silke Rönnebeck. (Dordrecht, the Netherlands: Springer, 2012): pp. 23-37.

PROBLEM SOLVING

Variable-Definition	Selecting or devising, as well as implementing, a mathematical strategy to solve problems arising from the task or context.
Level 0	Take direct actions, where the strategy needed is stated or obvious.
Level 1	Decide on a suitable strategy that uses the relevant given information to reach a conclusion.
Level 2	Construct a strategy to transform given information to reach a conclusion.
Level 3	Construct an elaborated strategy to find an exhaustive solution or a generalized conclusion; evaluate or compare strategies.

MODELING

Variable-Definition	*Mathematising* an extra-mathematical situation (which includes structuring, idealizing, making assumptions, building a model), or *making use* of a given or constructed model by *interpreting* or validating it in relation to the context.
Level 0	Either the situation is purely intra-mathematical, or the relationship between the real situation and the model is not needed in solving the problem.
Level 1	Interpret and infer directly from a given model; translate directly from a situation into mathematics (for example, structure and conceptualize the situation in a relevant way, identify and select relevant variables, collect relevant measurements, make diagrams).
Level 2	Modify or use a given model to satisfy changed conditions or interpret inferred relationships; or choose a familiar model within limited and clearly articulated constraints; or create a model where the required variables, relationships and constraints are explicit and clear.
Level 3	Create a model in a situation where the assumptions, variables, relationships and constraints are to be identified or defined, and check that the model satisfies the requirements of the task; evaluate or compare models.

COMMUNICATION

Variable-Definition	Decoding and *interpreting* statements, questions and tasks; including *imagining* the situation presented so as to *make sense* of the information provided; *presenting and explaining* one's work or reasoning.
Level 0	Understand a short sentence or phrase relating to a single familiar concept that gives immediate access to the context, where it is clear what information is relevant, and where the order of information matches the required steps of thought.
Level 1	Identify and extract relevant information. Use links or connections within the text that are needed to understand the context and task, or cycle within the text or between the text and other related representation/s. Any constructive communication required is simple, but beyond the presentation of a single numeric result.
Level 2	Use repeated cycling to understand instructions and decode the elements of the context or task; interpret conditional statements or instructions containing diverse elements; or actively communicate a constructed description or explanation.
Level 3	Create an economical, clear, coherent and complete description or explanation of a solution, process or argument; interpret complex logical relations involving multiple ideas and connections.

REPRESENTATION

Variable-Definition	*Interpreting*, translating between, and *making use* of given representations; *selecting* or *devising* representations to capture the situation or to present one's work. The representations referred to are depictions of mathematical objects or relationships, which include equations, formulae, graphs, tables, diagrams, pictures, textual descriptions and concrete materials.
Level 0	Directly handle a given representation, for example going directly from text to numbers, reading a value directly from a graph or table, where minimal interpretation is required in relation to the situation.
Level 1	Select and interpret one standard or familiar representation in relation to a situation.
Level 2	Translate between or use two or more different representations in relation to a situation, including modifying a representation; or devise a simple representation of a situation.
Level 3	Understand and use a non-standard representation that requires substantial decoding and interpretation; or devise a representation that captures the key aspects of a complex situation; or compare or evaluate representations.

As can be seen, PISA developed a four-level rubric for each of these competencies. A good way to interpret the PISA score levels across the PISA demands can be derived from how PISA built clusters of items to form the basis for their proficiency scale on the PISA examination. A score of 0 means that the demand is negligible. A score of 1 means the demand on the student is limited to *reproduction*. Reproduction includes using routine procedures, computations and representations. A score of 2 steps up the demand to requiring students to make *connections*. Connections include linking real world and mathematical representations and structures, standard problem solving, translation and interpretation and use of multiple well-defined methods. Finally, a level 3 requires *reflection* in addition to meeting all the lower level demands. Reflection includes complex problem solving and posing, insight, an original mathematical approach, multiple complex methods and generalization.

The CCSSM has eight Standards for Mathematical Practice that characterize mathematical expertise. These Practice Standards are closely related to the PISA competencies; however, the Practices describe student actions and are not readily applied to texts or problems isolated from the students. Rubrics for applying the Practices to texts and problems have yet to be developed. This, along with the high regard in which the PISA classification system is held, were the reasons why the PISA rubric was chosen for these practice competencies.[8] This methodology was developed by the co-chairs of the Panel working with two other members of NCEE's Technical Advisory Committee, James Pellegrino of the University of Illinois at Chicago and Joan Herman of UCLA.

The review process was organized to allow two panelists to examine what had been collected from each course with panelists coding each chapter in each text (that was assigned on the syllabus) and each question on each exam/assignments, marking which areas of content were covered at what grade level band and ranking each competency at one of four PISA levels (0, 1, 2 and 3). In addition to coding what mathematics was found in each course, a set of prerequisite mathematics that students would need in order to succeed in the mathematics courses was also developed. It leaned largely on a map of dependencies in the CCSSM developed by Jason Zimba, a member of the CCSSM leadership group, that illustrated how each standard was built on one or more other standards. This exercise was limited to the required mathematics courses as it was assumed that math was not generally taught in the subject matter courses and consequently students would have to know that math before coming to class. In other words, it was decided that the mathematics in the subject matter classes was in fact the prerequisite mathematics for these courses.

At the same time there was no designation of prerequisites for the PISA proficiencies identified in both the mathematics and subject matter courses as they are not seen as developmental characteristics, but as normative classifications associated with the cognitive demands of particular tasks.

A total of 20 mathematics courses and 43 introductory program courses were included in the analysis. There are fewer mathematics courses, as many of the mathematics courses are the first required mathematics course for multiple program areas. In addition, there are a few programs with no required mathematics course. Finally, not all of the colleges offered every program and not every college sent course materials for each program they do offer.

[8] See Mogens Allen Niss, Ross Turner, John Dossey and Werner Blum, "Using Mathematical Competencies to Predict Item Difficulty in PISA: A MEG Study" in *Research on PISA: Research Outcomes of the PISA Research Conference of 2009*, edited by Manfred Prenzel, Mareike Kobarg, Katrin Schöps and Silke Rönnebeck. (Dordrecht, the Netherlands: Springer, 2012): pp. 23-37 for a discussion of this rubric. An extension of OECD's thinking on mathematical literacy, in general, and modeling, in particular, can be found in their recently released frameworks for PISA 2012 (OECD, *PISA 2012 Assessment and Analytical Framework: Mathematics, Reading, Science, Problem Solving and Financial Literacy*, (Paris: OECD Publishing, 2013): pp. 23-58).

Below is a chart of introductory courses by program area of what was included in the study:

101 COURSES

Program	Number of Courses
Accounting	7
Autotechnology	3
Biotech/Electrical Technology	2
Business	4
Computer Programming	2
Criminal Justice	7
Early Childhood Education	7
Information Technology	5
Nursing	6
Total	**43**

The 20 mathematics courses were divided, for analytical purposes, into four broad categories. We collected materials for the six different College Algebra courses. Taken together they comprised the required Mathematics course for about 50 percent of the college programs that were studied.

MATHEMATICS COURSES

Type of Course	Number of Courses
College Algebra	6
Statistics	2
General Mathematics	7
Program-Specific Mathematics	5
Total	**20**

The analyses then proceeded to address three key questions:

- What mathematics is found in the introductory program courses in nine highly popular and diverse programs found in community colleges and in the mathematics courses typically required for those programs?
- What mathematics is needed to prepare for the initial required mathematics courses of these programs?
- What are the mathematical competency demands found both in the initial required mathematics courses and the introductory program courses of these programs?

This report is organized in the following way. We provide a detailed review of the findings, looking at the mathematics in the introductory program courses (referred to as the "101 courses") in each of the nine programs examined, then at the first required mathematics courses for each program area followed by the prerequisite mathematics for the required math courses. In each group of courses, we look at these findings overall, noting any differences among programs when they occur. These findings are followed by conclusions that review the major findings and their implications for our nation's schools.

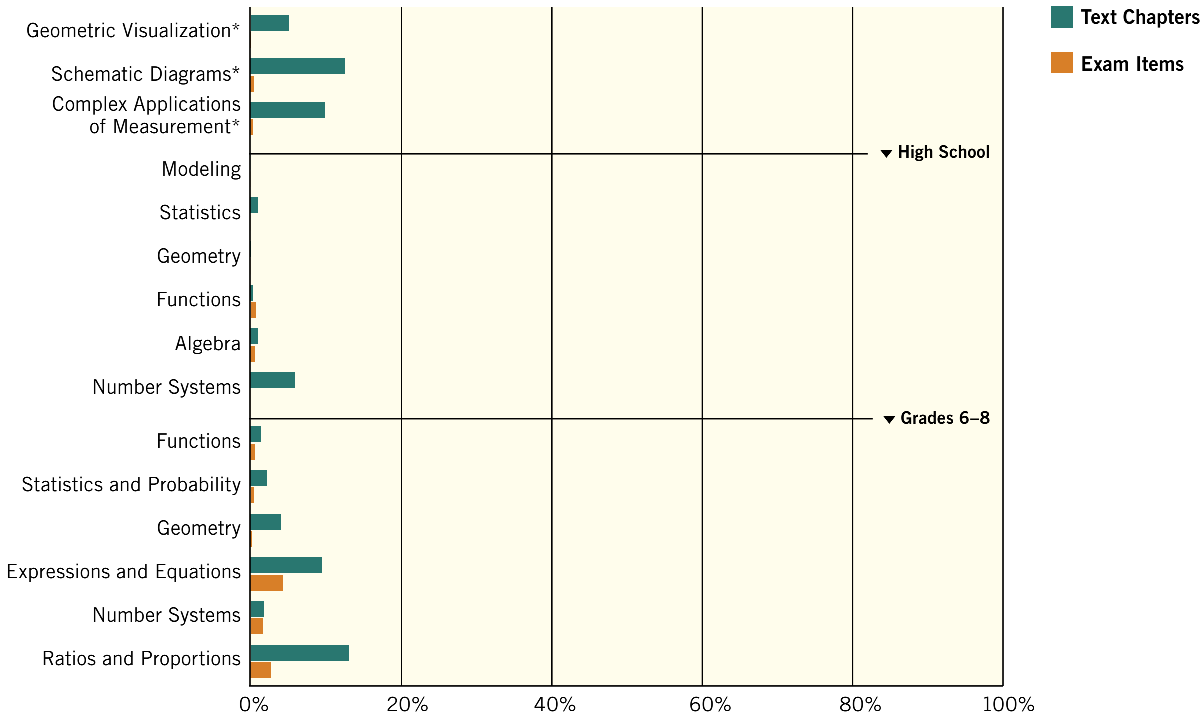
IV. FINDINGS

Mathematics Content in 101 Courses

THE INTRODUCTORY COURSE in each of the programs was analyzed to determine the mathematics necessary to succeed in the course. The panel observed and rated two different kinds of evidence: required reading, including the textbook(s), and graded final examinations and midterms and/or graded assignments having received samples of A, B and C grade work from college faculty.

For the textbook analysis, panel members searched for the mathematics that was needed to understand the content of the chapter. As described in the methodology section above, the panelists identified the topic and grade span of the mathematics using the CCSSM clusters. The percent of chapters in which mathematics was required in order to grasp the content being studied was selected as an overall indicator of how important mathematics is for comprehending a text and as a way to level the playing field between texts with 30 short chapters as against others with 6 long chapters.

CHART 1 *Average Percent of Text Chapters and Exam Items Containing CCSSM Domains for All Subject Matter (101) Courses*

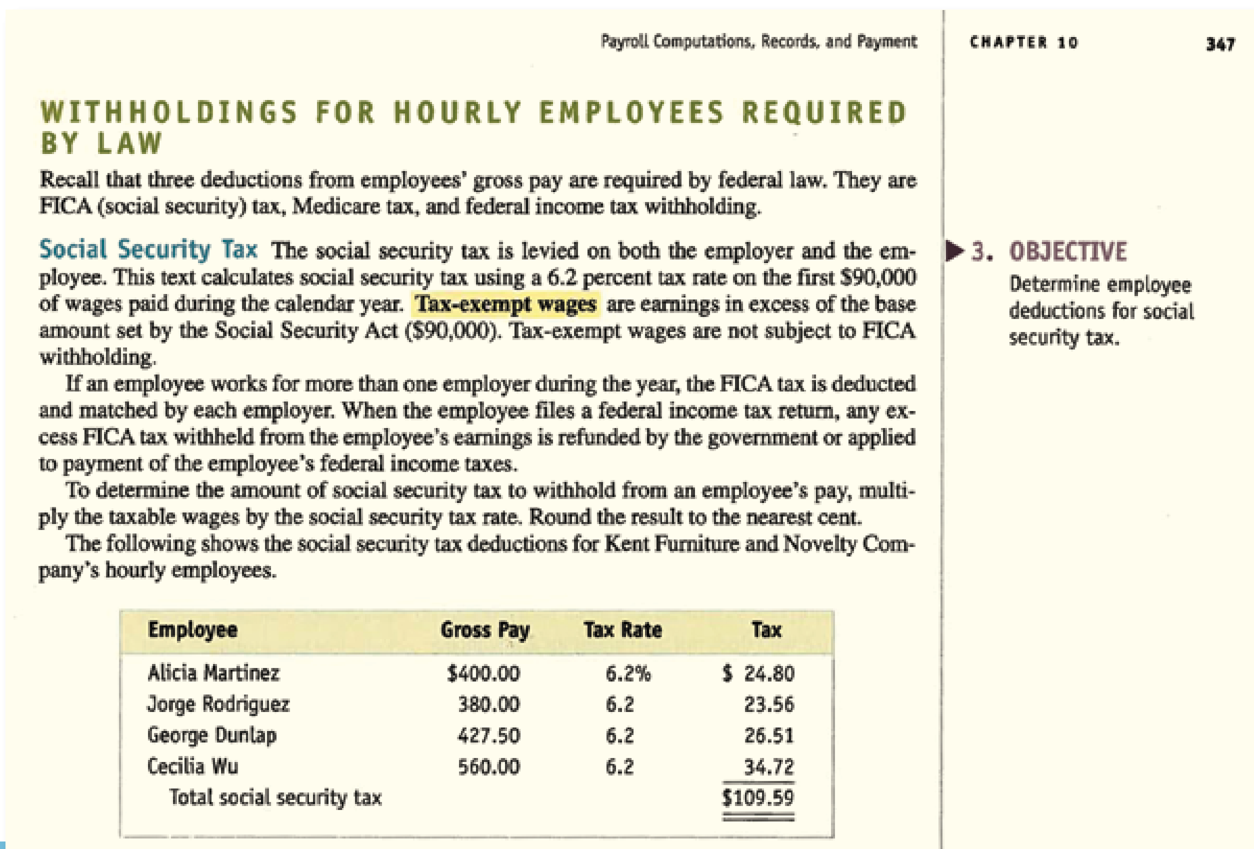


**Non-CCSSM domains added by the panel.*

The data from the analysis of textbooks across programs gives a clear answer to the question of what is required: middle school mathematics, especially ratios, proportionality, expressions and simple equations as described in the CCSSM, and three mathematical proficiencies not found in the CCSSM nor in typical school mathematics curricula: complex applications of measurement, schematic diagrams and geometric visualization. Complex applications of measurement, while essentially arithmetic, is much more demanding than what is typically expected of elementary students when they are taught measurement. It involves measurements with complex values, like those in medical dosages or medical laboratories. Texts in some classes also included schematic diagrams that use symbols rather than pictures to represent information. Geometric visualization, which includes 2D imaging of 3D objects, was also observed in many of the technical texts. While proportionality and these new areas are the areas of content with the most representation, their presence is still modest at about 10-13 percent in each area across the program areas (see Chart 1, previous page). This is not entirely surprising since these are subject matter courses, not mathematics courses.

While the texts in these courses provide one indicator of what counts when it comes to mathematics, it might also be argued that what is most consequential is what happens on exams, where the rubber hits the road. And here almost no mathematics appears in the 101 courses analyzed (see Chart 1, previous page). What little mathematics one finds in the exams involves simple middle school mathematics: a few substitutions into given formulae, but no algebraic manipulation; interpreting a few graphs and charts, but no analysis (see Figure 1, below). This type of math showed

FIGURE 1 ***Interpreting graphs and charts without analysis, Price, Haddock and Brock,*** College Accounting, *(New York: McGraw-Hill/Irwin, 2007), p. 347*



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WITHHOLDINGS FOR HOURLY EMPLOYEES REQUIRED BY LAW

Recall that three deductions from employees' gross pay are required by federal law. They are FICA (social security) tax, Medicare tax, and federal income tax withholding.

Social Security Tax The social security tax is levied on both the employer and the employee. This text calculates social security tax using a 6.2 percent tax rate on the first $90,000 of wages paid during the calendar year. **Tax-exempt wages** are earnings in excess of the base amount set by the Social Security Act ($90,000). Tax-exempt wages are not subject to FICA withholding.

If an employee works for more than one employer during the year, the FICA tax is deducted and matched by each employer. When the employee files a federal income tax return, any excess FICA tax withheld from the employee's earnings is refunded by the government or applied to payment of the employee's federal income taxes.

To determine the amount of social security tax to withhold from an employee's pay, multiply the taxable wages by the social security tax rate. Round the result to the nearest cent.

The following shows the social security tax deductions for Kent Furniture and Novelty Company's hourly employees.

Employee	Gross Pay	Tax Rate	Tax
Alicia Martinez	$400.00	6.2%	$ 24.80
Jorge Rodriguez	380.00	6.2	23.56
George Dunlap	427.50	6.2	26.51
Cecilia Wu	560.00	6.2	34.72
Total social security tax			$109.59

▶ 3. OBJECTIVE

Determine employee deductions for social security tax.

up in, at most, four percent of exam items. This pattern of little or no mathematics in first year examinations is true across the board in the areas of study analyzed. The areas of study that had the most mathematics on the examinations were Accounting and Computer Programming, with about 10 percent of Accounting examination items using middle school proportion and ratio and number systems and Computer Programming using middle school expressions and equations in 43 percent of exam items.[9] And though we searched far and wide for high school mathematics of any sort to make its way to these first year community college exams its appearance was the exception rather than the rule. So along this dimension it is not just that Algebra II mathematics is nowhere to be found, Algebra I mathematics is similarly absent.

Given the importance of middle school proportionality found in the texts across all the programs, a finer grained analysis of the content corresponding to the *cluster* level of the CCSSM was conducted for middle school mathematics, algebra and functions (see Chart 2, below).

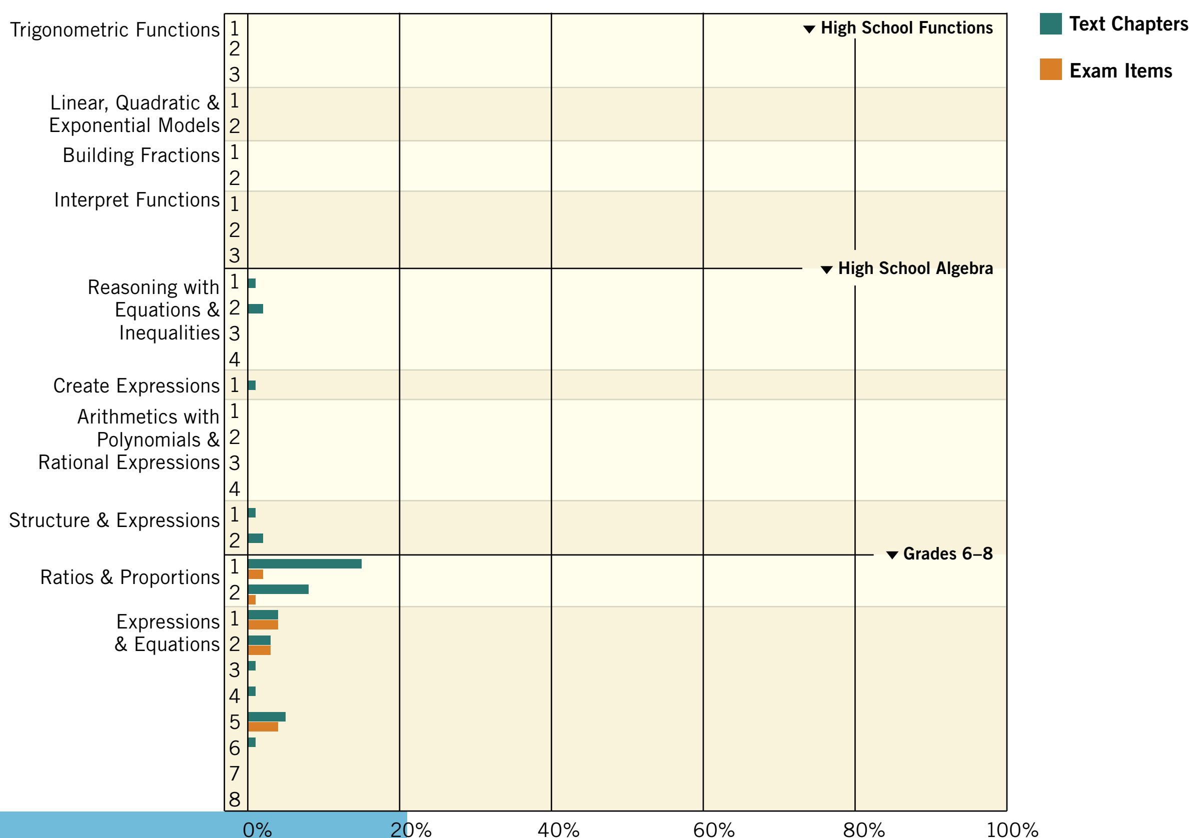
Our purpose was to determine exactly what areas of proportionality are emphasized and what aspects of algebra and functions are employed in these courses. There are two CCSSM clusters for proportionality:

1. *Understand ratio concepts and use ratio reasoning to solve problems.*

2. *Analyze proportional relationships and use them to solve real-world and mathematical problems.*

[9] Charts for each of the programs of study capturing the math found in texts and tests can be found in Appendix C.

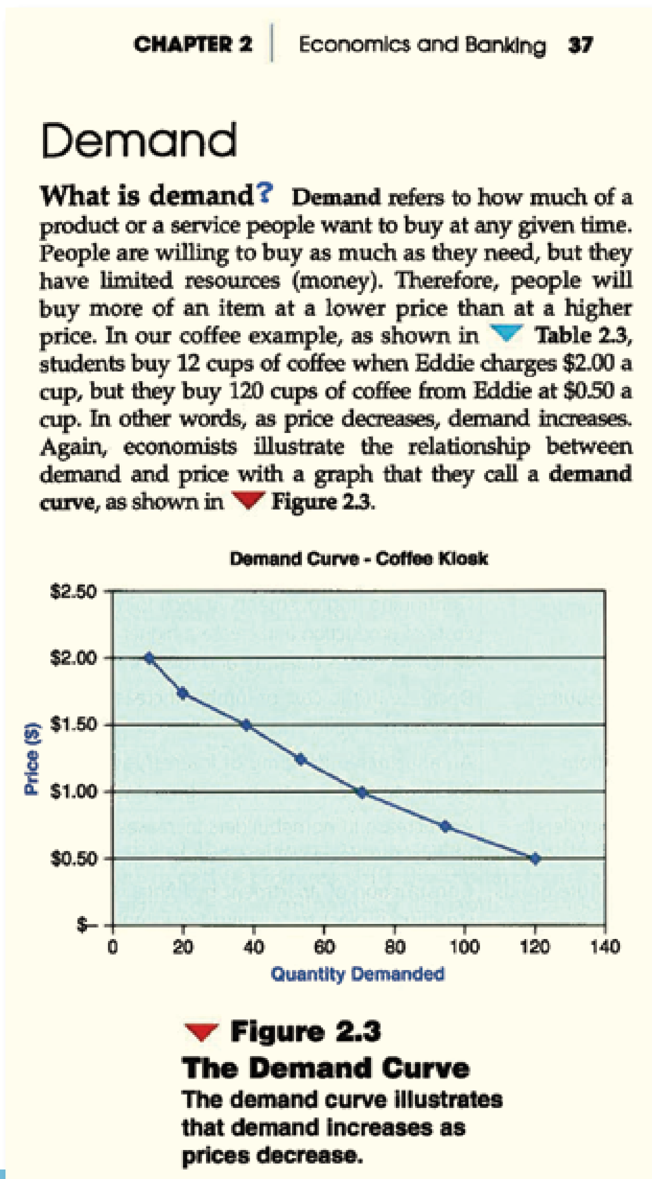
CHART 2 *Average Percent of Text Chapters and Exam Items Containing Grades 6-8 Standards for Ratios, Proportions, Expressions and Equations and High School Standards for Algebra and Functions for All Subject Matter (101) Courses**



**The column of numbers refers to the CCSSM cluster of standards within each domain specified in Appendix B.*

Our analysis found that both dimensions of ratio and proportion are present but that there is virtually no high school mathematics on the exams and very little in the texts. When mathematics is present in the texts, equations are not solved, quadratics are absent, and functions are present but not named or analyzed, just treated as formulae. Texts challenge students to apply middle school rate, proportionality and percent concepts to situations related to the content of the major. The concepts often arise in measurement and data contexts that include explanatory text illustrated with charts, graphs and tables (see Figures 2-6 for examples). Students do not have to perform algebraic manipulations nor construct graphs or tables; they do have to interpret the quantitative relationships in tables, graphs, and formula to make full sense of the text. The area of high school content with the highest representation in the texts, Number Systems, is found in six percent of the text chapters.

FIGURE 2 ***Functions present but not used,*** *Solomon, Poatsy and Martin,* Better Business, *(Upper Saddle River, NJ: Prentice Hall, 2010), p. 37*



CHAPTER 2 | Economics and Banking **37**

Demand

What is demand? **Demand** refers to how much of a product or a service people want to buy at any given time. People are willing to buy as much as they need, but they have limited resources (money). Therefore, people will buy more of an item at a lower price than at a higher price. In our coffee example, as shown in **Table 2.3**, students buy 12 cups of coffee when Eddie charges $2.00 a cup, but they buy 120 cups of coffee from Eddie at $0.50 a cup. In other words, as price decreases, demand increases. Again, economists illustrate the relationship between demand and price with a graph that they call a **demand curve**, as shown in **Figure 2.3**.

Figure 2.3
The Demand Curve
The demand curve illustrates that demand increases as prices decrease.

FIGURE 3 ***Middle school mathematics in charts and graphs with explanatory text,*** *Siegel,* Essentials of Criminal Justice, ***(Belmont, CA: Wadsworth, Cengage Learning, 2011), p. 299***

CHAPTER 9 • **Punishment and Sentencing** **299**

Deterrence

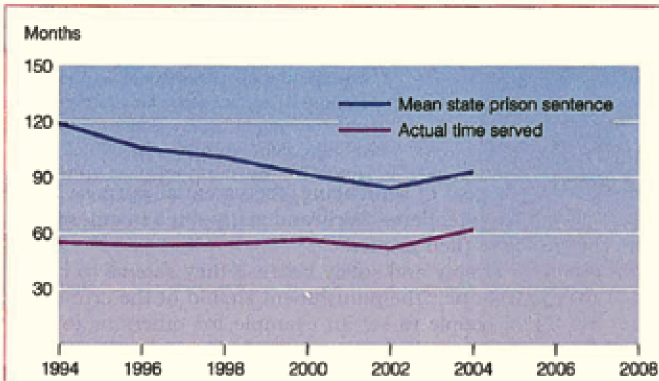
Deterrence is one of the most popular goals of sentencing. There are two types of deterrence: general and specific. According to the concept of **general deterrence**, people will be too afraid to break the law if they believe that they will be caught and punished severely. The more certain and severe the punishment, the greater the deterrent effect. However, punishment cannot be so harsh that it seems disproportionate and unfair. If it did, people would believe they had nothing to lose, and their crimes might escalate in frequency and severity. Thus, if the crime of rape were punished by death, rapists might be encouraged to kill their victims to dispose of the one person who could identify them at trial. Because they would already be facing the death penalty for rape, they would have nothing more to lose by committing murder as well.

general deterrence
A crime control policy that depends on the fear of criminal penalties. General deterrence measures, such as long prison sentences for violent crimes, are aimed at convincing the *potential law violator* that the pains associated with paying for the crime outweigh the benefits.

Some justice experts believe that the recent decline in the crime rate is a result of increasing criminal penalties. Once arrested, people have a greater chance of being convicted today than they did in the past. This phenomenon is referred to as "expected punishment," defined as the number of days in prison a typical criminal can expect to serve per crime.[6] Despite rising recently, expected punishment rates are actually still quite low because (a) crime clearance rates remain well under 50 percent, (b) many cases are dropped at the pretrial and trial stages (nolle prosequi), and (c) about one-third of convicted felons are given probationary rather than prison sentences.

Take the crime of burglary. About 2 million burglaries are reported to the police each year, about 200,000 burglars are arrested, 100,000 are convicted, and about 40,000 are sent to prison. Therefore, for every 50 reported burglaries, only one burglar is incarcerated. (Keep in mind that some burglars commit many crimes per year, so we are not talking about 2 million individual burglars but 2 million burglaries!) Such inefficiency limits the deterrent effect of punishment.

Because the justice system is still inefficient, the general deterrent effect of punishment is less than desired. The percentage of convicted offenders who now receive a prison sentence has actually declined during the past decade, and the estimated average prison sentence received by violent felony offenders in state courts decreased from nearly 10 years in 1994 to about 7 1/2 years today (Figure 9.1). If this trend continues, the deterrent effect of punishments may decline, and crime rates may increase as a result. Note, however, that the actual time served per sentence has increased somewhat, meaning that inmates are spending more of their sentence behind bars before they are released. This may help neutralize the effect of lighter sentences.[7]



Prison Sentence vs. Time Served for Violent Felonies

Source: Matthew Durose and Patrick Langan, *Felony Sentences in State Courts, 2004* (Washington, D.C.: Bureau of Justice Statistics, 2007).

FIGURE 9.1

FIGURE 4 *Text with rote instructions for using a formula, Gray Morris,* Calculate with Confidence *(St. Louis, MO: Mosby Elsevier, 2010), p. 217*

STEPS FOR USE OF THE FORMULA

Now that we have reviewed the terms in the formula, let's review the steps for using the formula (Box 15-1) before beginning to calculate dosages using the formula.

BOX 15-1 Steps for Using the Formula

1. Memorize the formula, or verify the formula from a resource.
2. Place the information from the problem into the formula in the correct position, with all terms in the formula labeled correctly, including "x.".
3. Make sure that all measures are in the same units and system of measure; if not, a conversion must be done *before* calculating the dosage.
4. Think logically, and consider what a reasonable amount to administer would be.
5. Calculate your answer, using the formula $\frac{D}{H} \times Q = x$.
6. Label all answers—tabs, caps, mL, etc.

FIGURE 5 *Middle school mathematics with explanatory steps for using a formula, taken from a computer programming final exam*

PROGRAMMING I with C++
FINAL EXAM

1. Write a program to calculate the number of slices that may be taken from a pizza of a given diameter. Input the diameter from the the user.
   - Each slice should have an area of 14.125 inches.
   - To calculate the number of slices, simply divide the area of the pizza by 14.125.
   - The area of the pizza is calculated with this formula:
   - Area = *PI* r^2.
   - 3.14159 can be used for the value of *pi*. The variable *r* is the radius of the pizza. Divide the diameter by 2 to get the radius.

```
What is the diameter of the pizza?  12
Cut the pizza into 8 slices.
```

FIGURE 6 *Middle school ratio and proportion, taken from Nursing 101 final exam*

3. The doctor orders 1,800 mL D5W to be infused over 24 hours. The nurse sets the infusion pump at:

a. 3 mL/hr.
b. 7.5 mL/hr.
c. 30 mL/hr
d. 75 mL/hr.

The three areas of content not addressed in the CCSSM (complex applications of measurement, schematic diagrams and geometric visualization) were observed most often in Autotechnology, Nursing and Technology texts, and sometimes on the exams for these courses. The Nursing and Biotechnology texts, in particular, require Complex Applications of Measurement (see Figure 7, below). Autotechnology texts had substantial geometric visualization and reasoning in the use of complicated 2-D schematics of 3-D objects although there was little Euclidian geometry (see Figure 8, below). Schematic diagrams like flow charts and decision trees played important roles in some texts especially Computer Programming and Autotechnology (see Figure 9, below). These domains are more characteristic of how mathematics is used in career applications in contrast to the content that typically serve as the centerpieces of high school mathematics.

FIGURE 7 ***Complex Applications of Measurement, Seidman,*** Basic Lab Calculations for Biotechnology, *(San Francisco, CA: 2008), p. 361*

2. You have a cell culture flask containing 75 mL of a suspension of treated cells. You must remove 0.2 mL of the cell suspension to a tube and add 0.5 mL of trypan blue and 0.3 mL of sterile medium. You apply the diluted cells to a hemocytometer and count the viable and non-viable cells in the four corner squares. There are 226 clear cells and 25 blue cells. You do not count the center square because you have already counted enough cells.

a. What is the average number of viable cells per square?

b. What is the percent viability?

c. What is the inverse of the dilution?

d. What is the density of cells in the original cell culture dish?

e. How many cells are there altogether in the original 75 mL?

FIGURE 8 ***Geometric Visualization, Owen,*** Basic Automotive Service & Systems, *(Clifton Park, NY: Delmar, Cengage Learning, 2011), p. 80*

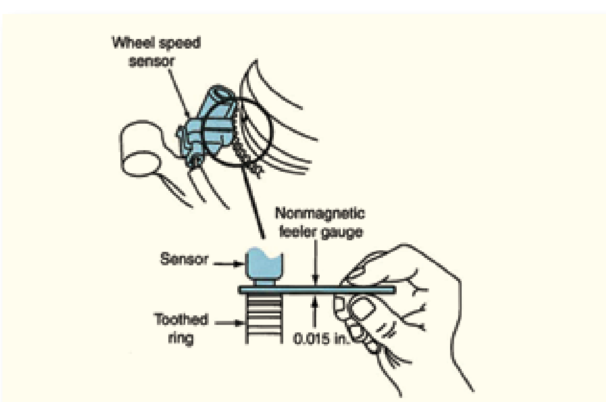
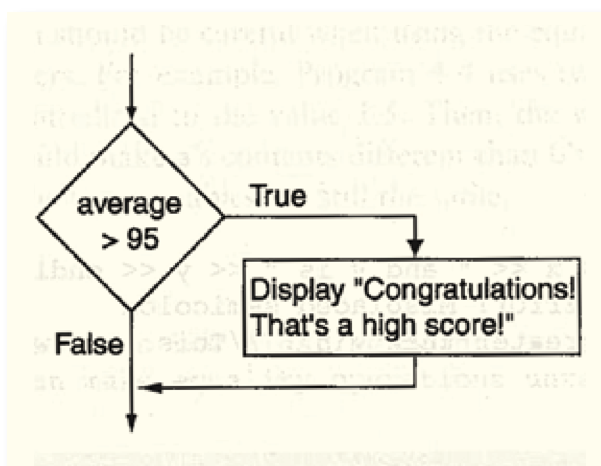


Figure 4-5 Measuring the air gap between the sensor and a tooth or trigger. A nonmagnetic (usually brass) feeler gauge is used to measure the air gap.

FIGURE 9 ***Schematic Diagram, Gaddis,*** Starting Out With C++, *(Boston, MA: Pearson, 2010), p. 167*



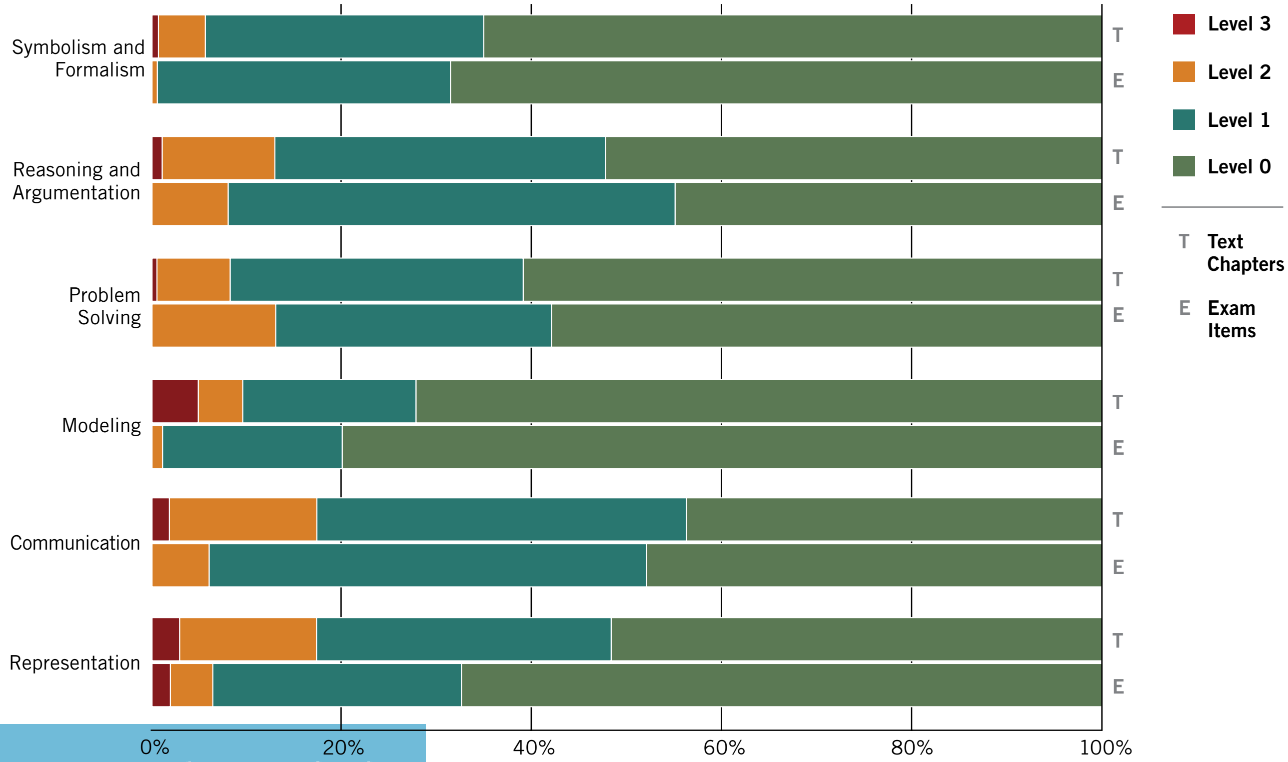
V. FINDINGS

Mathematics Practices in 101 Courses

WHILE MATHEMATICS CONTENT is one critical dimension of mathematics deserving of attention in any examination of college readiness, mathematics practices also deserve close scrutiny, as one doesn't exist without the other. Chart 3 (below) shows that across programs, the texts for 101 courses were modest in their demands for PISA proficiencies that gauge strategic competence and related higher order cognitive demands.[10] Almost 90 percent of the chapters that use mathematics use it in routine and simple ways, with PISA ratings of 0 or 1. Computer Programming and Biotechnology texts are rated considerably higher on most of the proficiency scales with 75 percent of the texts earning a 2 on communication and 52 percent rating a 2 or 3 on representation, but these are the glaring exceptions to the typical mathematics practice demands we found in entry level courses. On the course examination front, even more modest PISA expectations are to be found with closer to 95 percent of exam items rated 0 or 1. Here again Programming stands out with 33 percent of the exam items rated at 2 for communication, problem solving and reasoning and argumentation, but these courses are very much the exception to the rule. The proficiency with the highest percentage of 2 and 3 rankings across all programs was problem solving with 8 percent. This suggests that neither the texts nor the exams found in 101 courses require a high level of complex mathematical thinking.

[10] Program by program findings of PISA proficiencies can be found in Appendix D.

CHART 3 *Average Percent of Text Chapters and Exam Items Rated at Each PISA Level for All Subject Matter (101) Courses*



Sum of percent of ratings at each level for all courses which contain mathematics, divided by total number of courses which contain mathematics. Courses that do not include mathematics are not included in the numerator or denominator.

VI. FINDINGS

Discrepancies Between Texts and Exams in 101 Courses

THE GAP BETWEEN the mathematics on the exams and the mathematics in the texts presents a dilemma in determining the mathematics required for succeeding in these courses. The Panel understood this as a gap between what math textbook authors believed was integral to grasping the subject matter of a course and what math community college faculty either thought was essential to their curricular objectives or thought their students could master or both. One has to ask if it is critical for students to fully comprehend the text in order to succeed in the course. One way to interpret the difference in textbook evidence compared to the examination evidence is that there is a spectrum of possible "success" in a course: from doing just what is needed to pass the course to learning as much as possible. The tests give an indication of what a student who just wants to pass the course needs to know, while the textbooks indicate what someone who wants to take full advantage of the learning opportunity will want to comprehend. On this spectrum of success, the mathematics needed for success in 101 courses ranges from middle school mathematics down to almost none.

Given this, the Panel concluded that little or no high school mathematics was needed to do well on the examinations but that some middle school mathematics, in particular proportionality, and the three areas identified outside the CCSSM (complex applications of measurement, schematic diagrams and geometric visualization) were needed to comprehend the texts and fully participate in class. They also observed that teachers in 101 courses, judging by what is tested, do not require students to apply mathematics or to think mathematically even when the text for the course uses mathematics to explain an important idea.

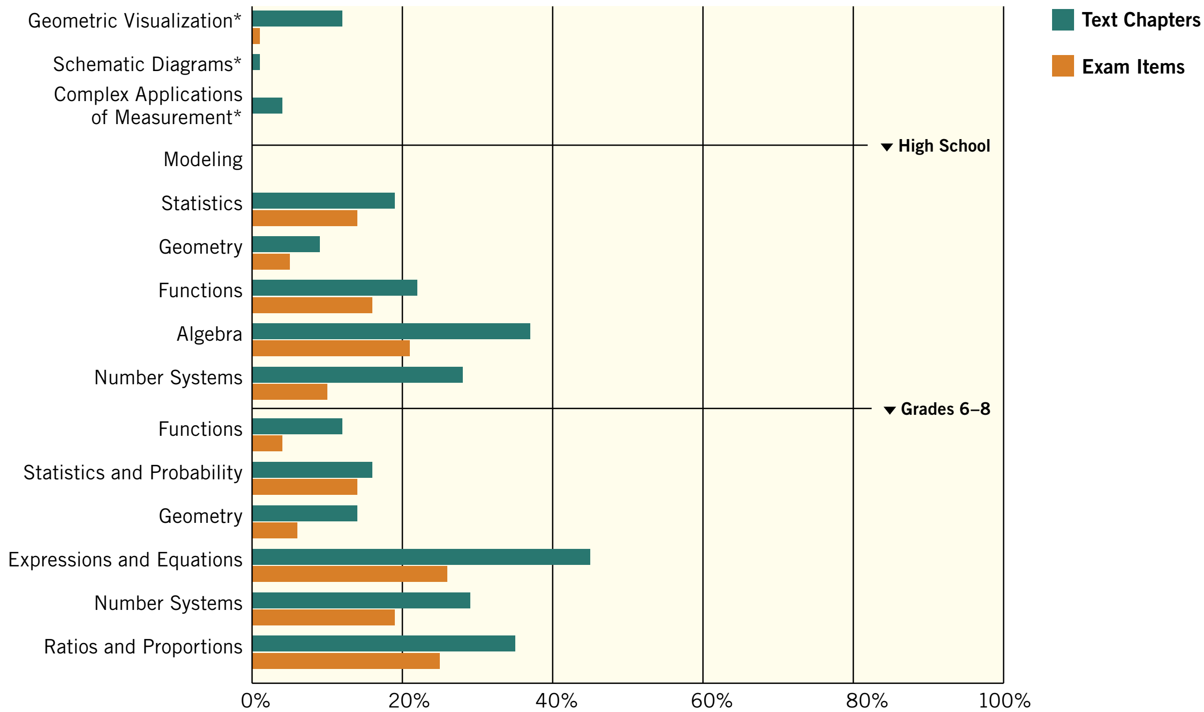
VII. FINDINGS

Mathematics Content and Practices in Mathematics Courses

THE TEXTS AND EXAMINATIONS across the mathematics courses, not surprisingly, find the various forms of mathematics in much greater abundance than what was found in the 101 courses. The content domains in the college mathematics courses are shown in Chart 4 (next page). Unlike the 101 courses, the mathematics texts and examinations of the mathematics courses parallel each other in content, for the most part. Since these are all required courses for one or another of the college programs in this study, it is surprising how well represented middle school mathematics is in the text chapters and examination items. Expressions and simple equations (nothing beyond linear in middle school), proportionality and rates, and the number system are particularly well represented, showing up in 27-49 percent of text chapters and 15-30 percent of exam items. High school mathematics is also well represented in the texts and in the examinations, with algebra and functions in particular represented in 45 and 27 percent of text chapters and 28 and 16 percent of exam items. However, two of the three areas of content identified in the 101 courses outside the CCSSM (complex applications of measurement and schematic diagrams) were seen at much lower levels in the mathematics courses than in the 101 courses. Geometric visualization was represented in about 11 percent of chapters but hardly at all on exams. In all of these findings each course is weighted according to the extent it is required by the 101 courses we studied. In this way if a single college algebra course is the required course for multiple programs of study it will carry greater weight than if it was required by only one.

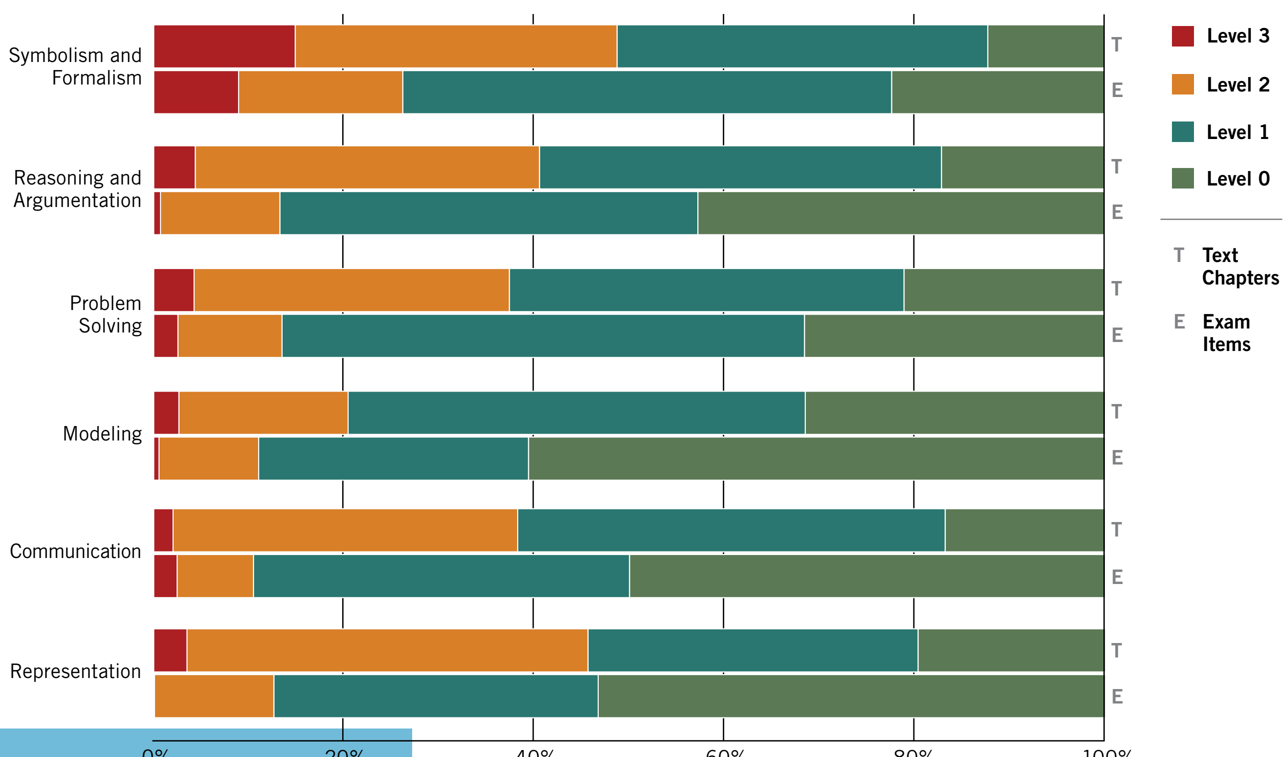
The PISA demands (Chart 5, next page) of the texts involved require students to think about the mathematics they are working with by making connections and reflecting on them at a level beyond the routine use of mathematics. This is represented by scores of 2 and 3 in over a third of the chapters in every dimension but modeling. The examinations, however, only had such demands (and scores of 2 and 3) for 10-15 percent of the items and this disjuncture held across five of the six PISA dimensions. For example, modeling had such demands in 11 percent of the items, problem solving at 14 percent and reasoning and argumentation at 15 percent. The PISA demands were highest for symbolism and formalism, and lowest for communication and modeling on the exams. A greater emphasis on communication and modeling while dialing back the traditional focus on formalism would better serve the programs of study that these prerequisite courses are designed to support. Overall, the PISA demands were substantially higher for the mathematics texts and exams than the 101 texts and exams.

CHART 4 *Average Percent of Text Chapters and Exam Items Containing CCSSM Domains for All Required Mathematics Courses*



**Non-CCSSM domains added by the panel.*
Mathematics courses that serve as the requirement for multiple programs of study are weighted to reflect this reality.

CHART 5 *Average Percent of Text Chapters and Exam Items Rated at Each PISA Level for All Required Mathematics Courses*



VIII. FINDINGS

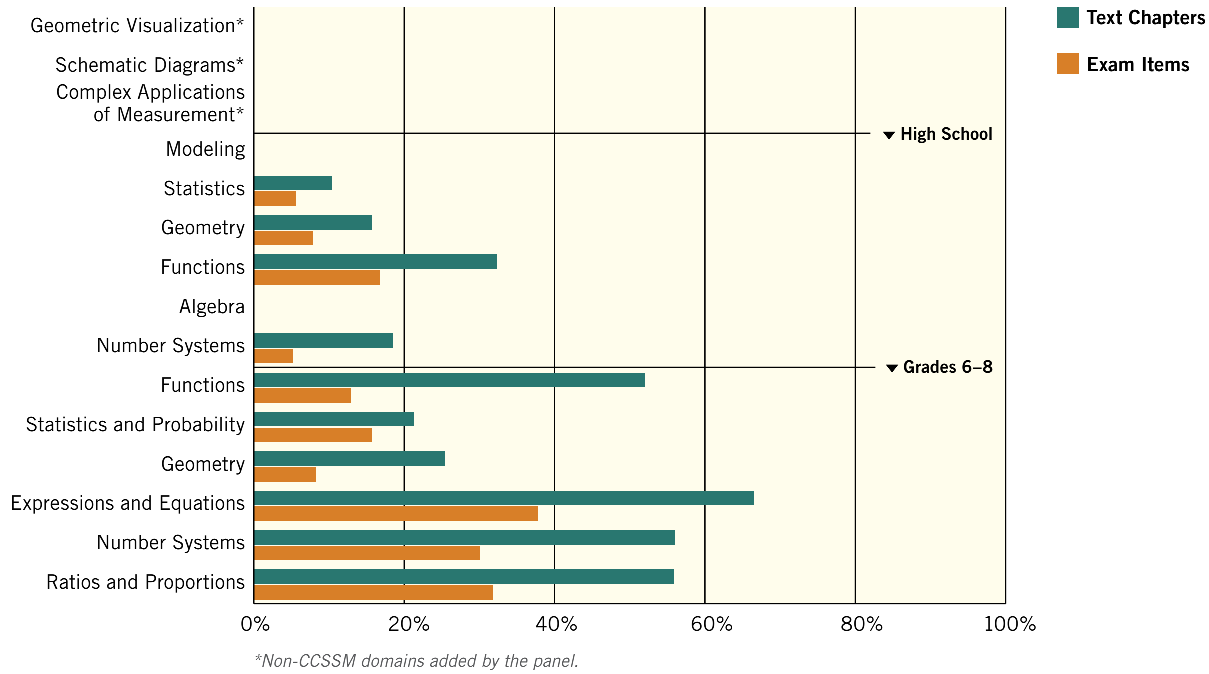
Mathematics Course Prerequisites

THE PREREQUISITE CONTENT needed to succeed in the initial required mathematics courses' textbooks is shown in Chart 6 (next page). Given the overlap of content in the college courses (Chart 4, previous page) with high school mathematics, it is no surprise that middle school mathematics plays a prominent role, and even more so in examinations than in texts. Proportionality, expressions, simple equations, number and functions from middle school stand out, as do some aspects of high school algebra and functions. Given that ratio, proportion, algebra and functions at the high school level encompass a broad range of content, in Chart 7 (next page) we have disaggregated these conceptual categories into their component parts.[11] Now we can see that the prerequisite high school mathematics is largely solving simple equations, solving equations graphically and interpreting functions. Thus one is drawn to the finding that in these four conceptual categories the mathematics that count as important for success in community college lie at the heart of Algebra I and not in Algebra II.

It might seem odd that in Chart 6 (next page) neither geometric visualization, schematic diagrams nor complex applications of measurement appear as prerequisites for any mathematics courses when these are important aspects of some community college mathematics courses, but this is nothing more than an artifact of the reality that these aspects of mathematics are not presently found in the dependencies for the CCSSM on which this data rested nor, for that matter, are they found in our schools. While there are aspects of number and geometry in the CCSSM and our schools that can serve as building blocks for geometric visualizations and the like, adequate preparation for such complex and demanding problems as students will encounter in community college demands much more as this is especially challenging work. Often requiring students to analyze multiple representations of complex machinery or social systems, this presents a more heightened reality than the simple "real world" problems they see in today's middle and high school classrooms. It requires reasoning that draws on a deep understanding of mathematics and represents a significant advance in the application of mathematics. So just as secondary school mathematics finds students moving from basic computation and procedure to the abstraction and conceptual sophistication of algebra it also ought to start providing students with experience with these more demanding forms of mathematical thinking and problem solving if they are to be ready for what awaits them in college.

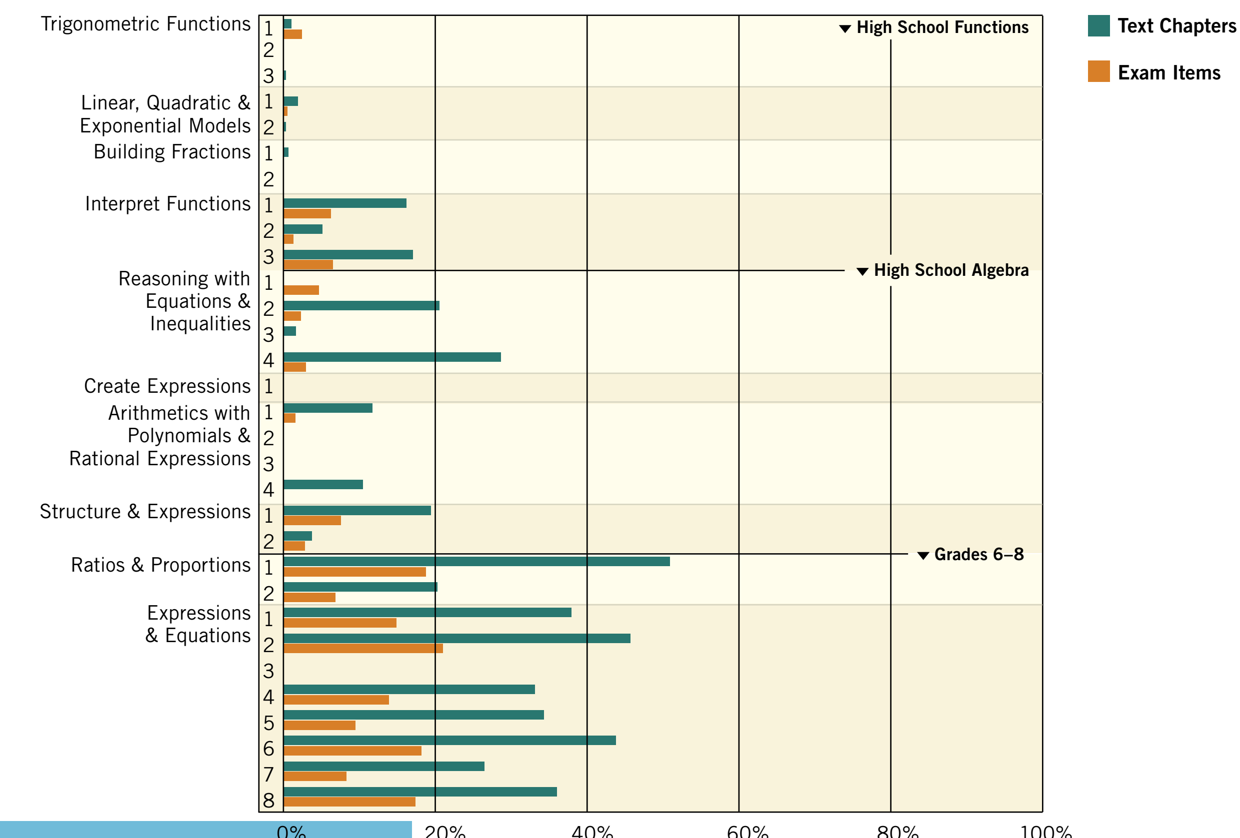
[11] A table of the numbered clusters that appear within each domain that is displayed in Chart 7 can be found in Appendix B.

CHART 6 *Average Frequency of Prerequisites Called for by All Required Mathematics Courses: Based on the Percent of Text Chapters and Exam Items that Utilize Each CCSSM Domain*



*Non-CCSSM domains added by the panel.

CHART 7 *Average Frequency that Grades 6-8 Standards for Ratios, Proportions, Expressions and Equations and High School Standards for Algebra and Functions Appear as Prerequisites for All Required Mathematics Courses: Based on the Percent of Text Chapters and Exam Items that Utilize Each CCSSM Domain**



For charts 7 and 8, mathematics courses that serve as the requirement for multiple programs of study are weighted to reflect this reality.
**The column of numbers refers to the CCSSM cluster of standards within each domain specified in Appendix B.*

IX. FINDINGS

By Type of Mathematics Course

THE FINDINGS IN CHARTS 4, 5, 6 and 7 are for all required mathematics courses. However, different programs and different colleges require different initial mathematics courses.[12] To provide a sense of these variations we move one level down and examine the main types of mathematics courses that populate the community colleges we studied – College Algebra, Statistics, General Mathematics and Program Specific Courses.

College Algebra

The content analysis of College Algebra (Chart 8, next page) turns up a significant measure of redundancy with middle and high school algebra and functions, and a few geometry topics. A closer look at the CCSSM clusters reveals that the emphasis of the College Algebra courses is on topics typically treated in traditional Algebra I and some of Algebra II. Trigonometric functions get little attention. The Panel did not find any additional content beyond CCSSM High School in the College Algebra courses.

The examinations mirror the texts in emphasis except that the texts pay more attention to number systems and middle school mathematics than the examinations do. This may be due to the fact that the college texts include some review chapters that are not tested.

The PISA demands (Chart 9, next page) for symbolism and formalism are substantial on both examinations and in texts. Ninety-four percent of texts scored at 2 or 3 and 58 percent of exam items scored at this level. Problem solving and representations also demand some higher orders of thinking, with 50 percent of text chapters scoring a 2 or 3 in problem solving and 57 percent of text chapters scoring a 2 or 3 on representation.

Statistics

The required statistics courses (Chart 10, page 28) are closely aligned with the CCSSM high school and middle school statistics and probability specifications, with about 65 percent of exam items and over 90 percent of text chapters covering high school statistics and probability and about 85 percent of the exam items and over 90 percent of the text chapters covering middle school statistics and probability. The college statistics courses also revisited middle school algebra, number systems and especially rates and proportionality.

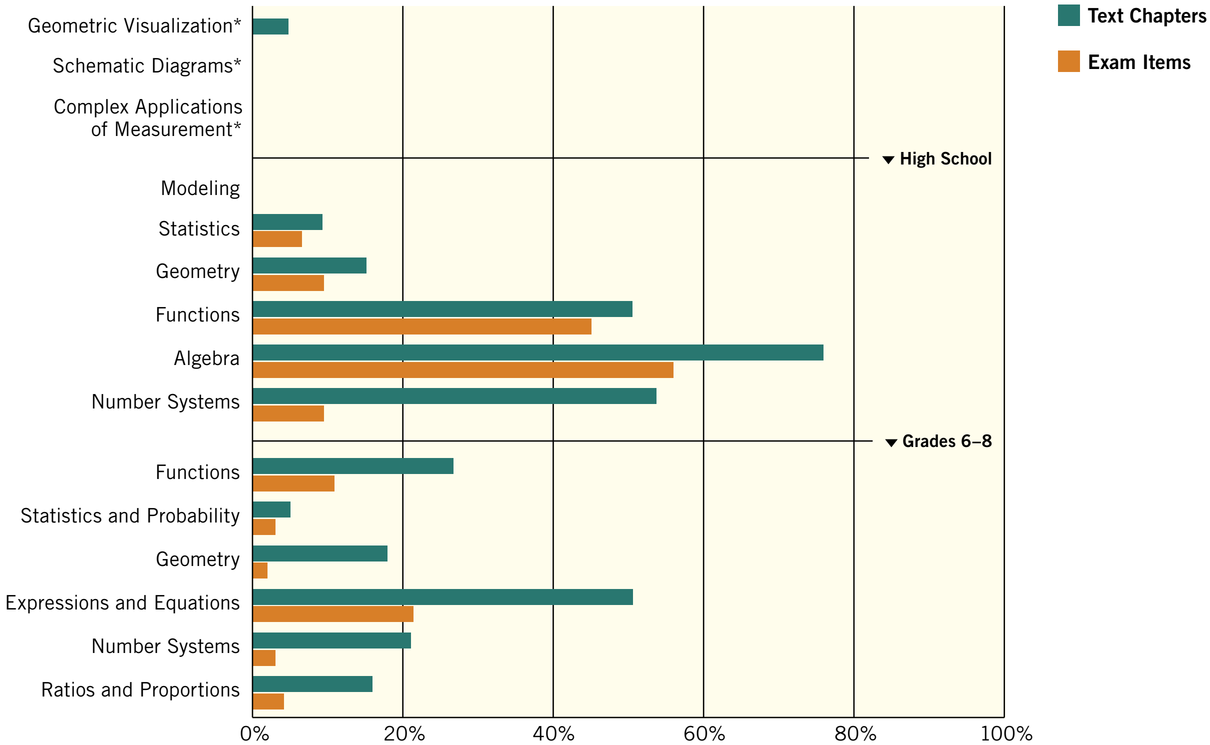
The PISA demands (Chart 11, page 28) were moderate in the texts for representations, communication, reasoning and symbolism, with 50 to 80 plus percent of text chapters scoring 2s on these proficiencies. There were almost no scores of 3. The tests were less demanding, with about 20 percent of test items scoring 2 on these proficiencies and no 3s present.

General Mathematics

General mathematics courses are a level below College Algebra, which was, itself, overlapping with high school mathematics (Chart 12, page 30). In the textbooks, middle school expressions and equations, ratio and proportion, and the number system predominate with some

[12] The distribution of required mathematics courses by program can be found in Appendix E.

CHART 8 *Average Percent of Text Chapters and Exam Items Containing CCSSM Domains for College Algebra Courses*



**Non-CCSSM domains added by the panel.*

CHART 9 *Average Percent of Text Chapters and Exam Items Rated at Each PISA Level for College Algebra Courses*

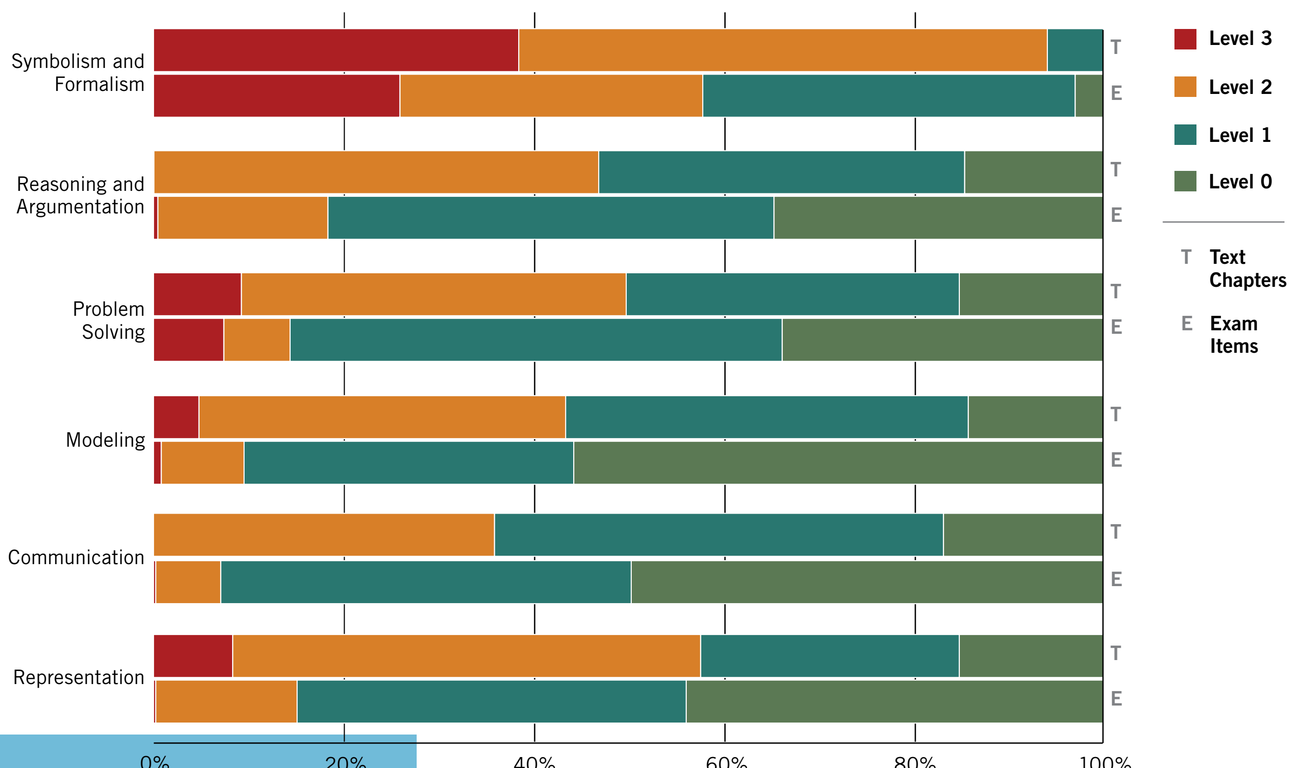
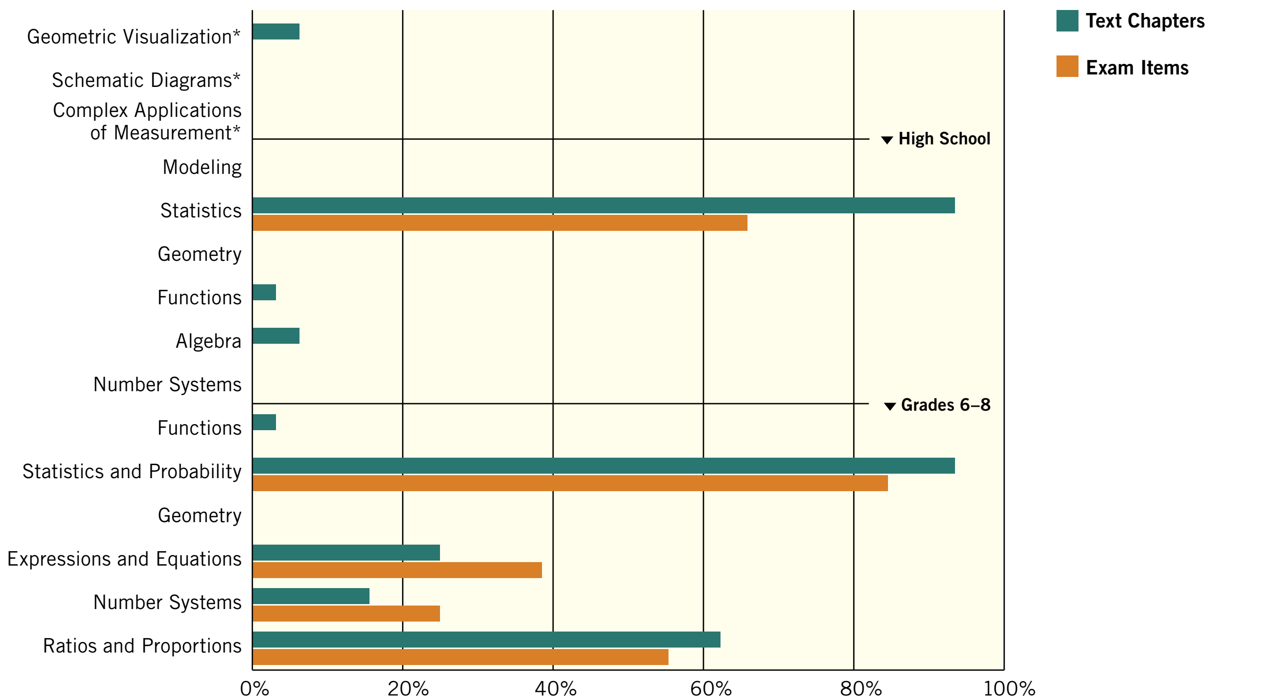
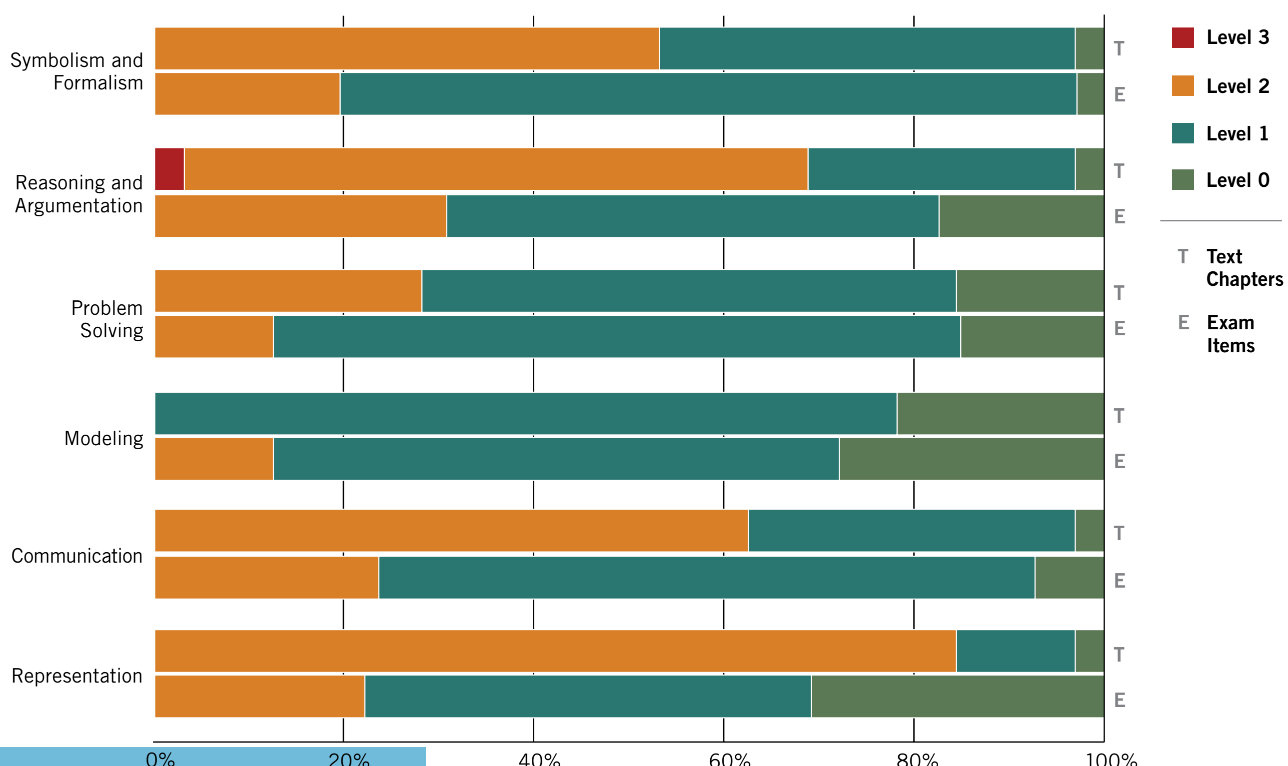


CHART 10 *Average Percent of Text Chapters and Exam Items Containing CCSSM Domains for Statistics Courses*



*Non-CCSSM domains added by the panel.

CHART 11 *Average Percent of Text Chapters and Exam Items Rated at Each PISA Level for Statistics Courses*



high school algebra and number systems. Middle school expressions and equations are represented in more than 50 percent of text chapters and middle school ratio and proportion is represented in about 40 percent of chapters. Also included are geometric visualizations outside the CCSSM but relevant to the courses in the major, with close to 20 percent of chapters including this concept. The exams reflected the texts with somewhat more weight allocated to middle school content rather than high school.

PISA demands were modest for the texts but negligible for the examinations, with roughly a third of the text chapters rated 2s and 3s across five of the six proficiencies, but only three to six percent of the exams rated similarly across five of the six proficiencies (Chart 13, next page).

Program Specific Courses

A few mathematics courses that we uncovered were specific to the program that required them. The samples included "Math for Biotechnology," "Medical Dosages (Nursing)" and "Business Math."[13] Like the other math courses, middle school mathematics figures significantly in the texts for these courses with some interesting additions. The course on medical dosage includes "complex applications of measurement," one of the content domains found in subject courses but not in the CCSSM. This likely reflects something of the career specific relevance one would expect of this mathematics course. The Early Childhood Education mathematics courses include "geometric visualization," another domain found in subject courses but not in the CCSSM.

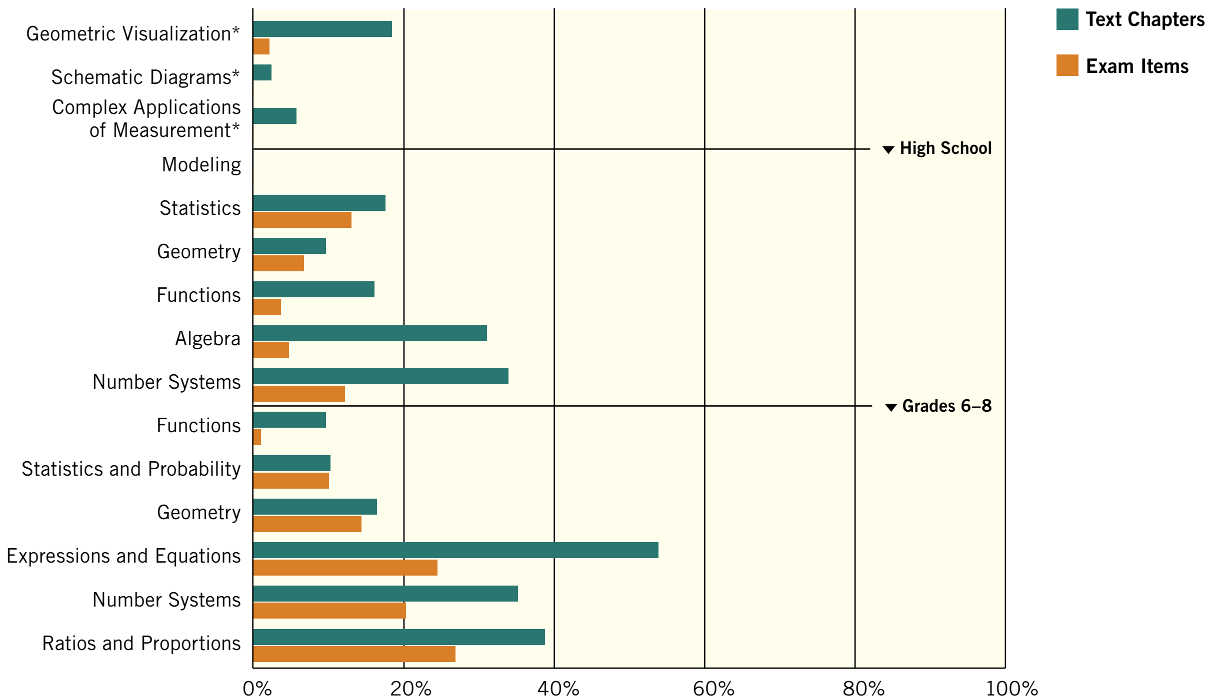
Only the Early Childhood mathematics courses had substantial PISA demands, with scores of 2 and 3. The other courses were very modest in PISA demand, usually 1 and 0. The oddity here is that math is nowhere to be found in the 101 Early Childhood courses sampled. The likely reason for this discontinuity is that the mathematics courses selected for Early Childhood programs are the mathematics courses designed for elementary school teachers.

Since these courses are typically designed with the major program in mind, or at least the referenced career in the course titles, one would expect some attention to relevance in mathematical focus. One would also expect mathematics geared to preparation levels typical of students in these majors, another kind of relevance. From this perspective, the focus of these courses is an indicator of what mathematics is relevant to these programs. Here the relevance of middle school mathematics is salient, as is the inclusion of non-traditional content in complex applications of measurement and geometric visualization. So here there is some measure of consistency between mathematics and the 101 courses, a healthy linkage typically not found in programs of study that have less control of the available mathematics offerings for their students and who find themselves to a much greater extent than they might like at the mercy of the priorities of their mathematics department colleagues and their view that if students "learn" a math concept its application will readily follow.

Of course, once the inspection of mathematics courses reaches this level of granularity with respect to the content of these courses the follow-on issue is, what are the prerequisite specifications at this same level? The answers can be found in the charts in Appendix G.

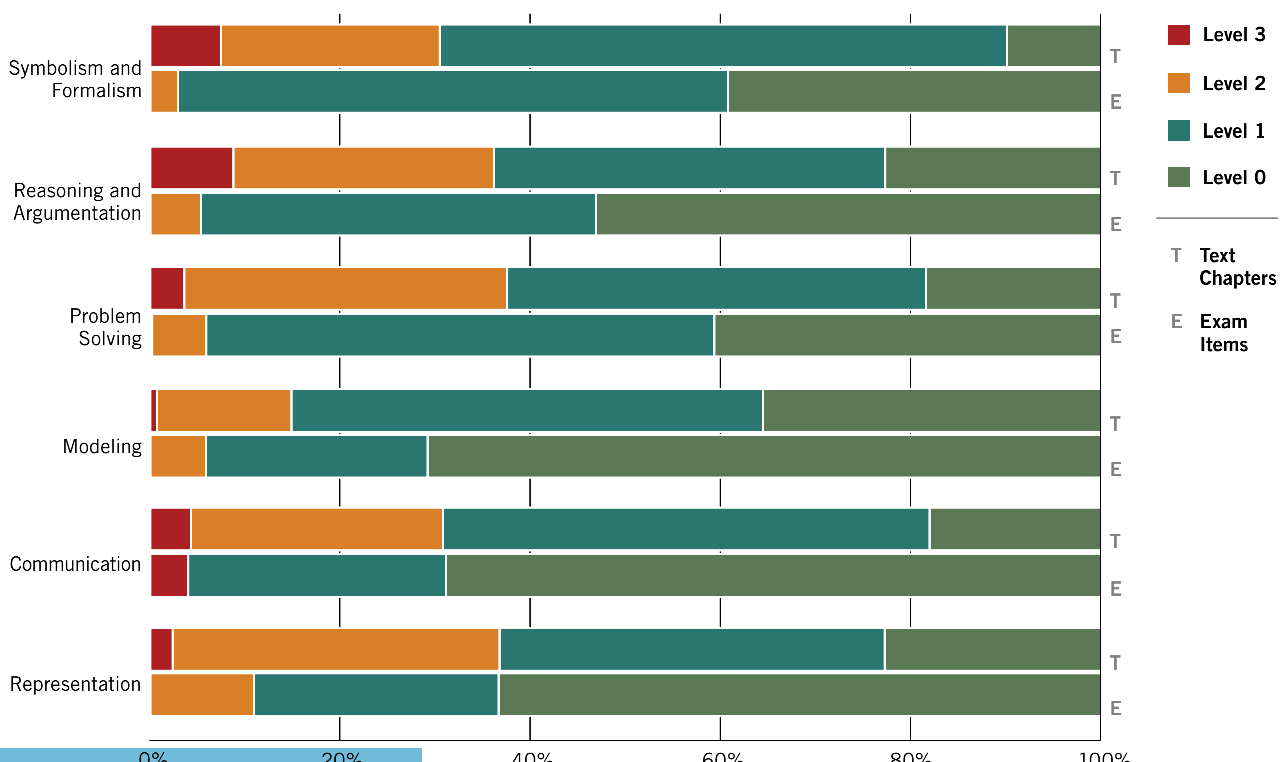
[13] See Appendix F for detailed charts.

CHART 12 *Average Percent of Text Chapters and Exam Items Containing CCSSM Domains for General Mathematics Courses*



*Non-CCSSM domains added by the panel.

CHART 13 *Average Percent of Text Chapters and Exam Items Rated at Each PISA Level for General Mathematics Courses*



X. FINDINGS

Mathematics in 101 Courses vs. Mathematics in Required Mathematics Courses

WHEN THE EVIDENCE of what transpires in the mathematics courses is set against the evidence of what transpires in the 101 courses that depend on them, there appears to be a significant mismatch between these two component parts on our community college campuses. Here we consider this issue in greater detail.

In Chart 14 (next page), the horizontal scale shows the difference between the percent of 101 course chapters including mathematics from the domain indicated minus the percent of chapters including that domain in the text from the required mathematics course. For example, if a criminal justice text included high school algebra in 10 percent of its chapters, and the required mathematics course included high school algebra in 65 percent of its chapters, then Chart 14 would show -55 percent for criminal justice. A negative percent indicates heavier emphasis on the domain of mathematics in the mathematics course than in the subject course.

Given that mathematics courses will emphasize mathematics more than 101 subject courses, it is surprising to see several areas of mathematics emphasized more in the subject courses than in the mathematics courses. The three domains not found in the CCSSM (geometric visualization, complex applications of measurement and schematic diagrams) are also found little, if at all, in the community college mathematics courses. In a few fields, Accounting and Computer Programming, the 101 subject courses included more middle school proportionality and middle school expressions and simple equations than did the mathematics course, but these are the exception to the rule.

The story with respect to examinations (Chart 15) is similar. Overall, high school algebra gets far more attention in the initial required mathematics courses than in the 101 subject courses. This is not surprising considering the overlap in content between high school courses and community college courses but it also suggests that colleges are teaching more advanced mathematics than students need for their program studies. While this can be justified to some degree by the argument that all students should have a basic grounding in mathematics irrespective of their specific occupational needs and those students in a transfer program going on to a four-year college may need more mathematics by the end of their second year of college than those in two-year occupational degree and certificate programs, it is not at all clear that the topics covered by the typical College Algebra course make sense for most students, especially if a student is required to pass College Algebra in order to get a degree or certificate from a two-year occupational program, for all the reasons suggested throughout this report.

CHART 14 *Discrepancy Between Average Percent of Text Chapters Containing Mathematics Content in 101 Courses and Average Percent of Text Chapters Containing Mathematics Content in Required Mathematics Courses, All Programs*

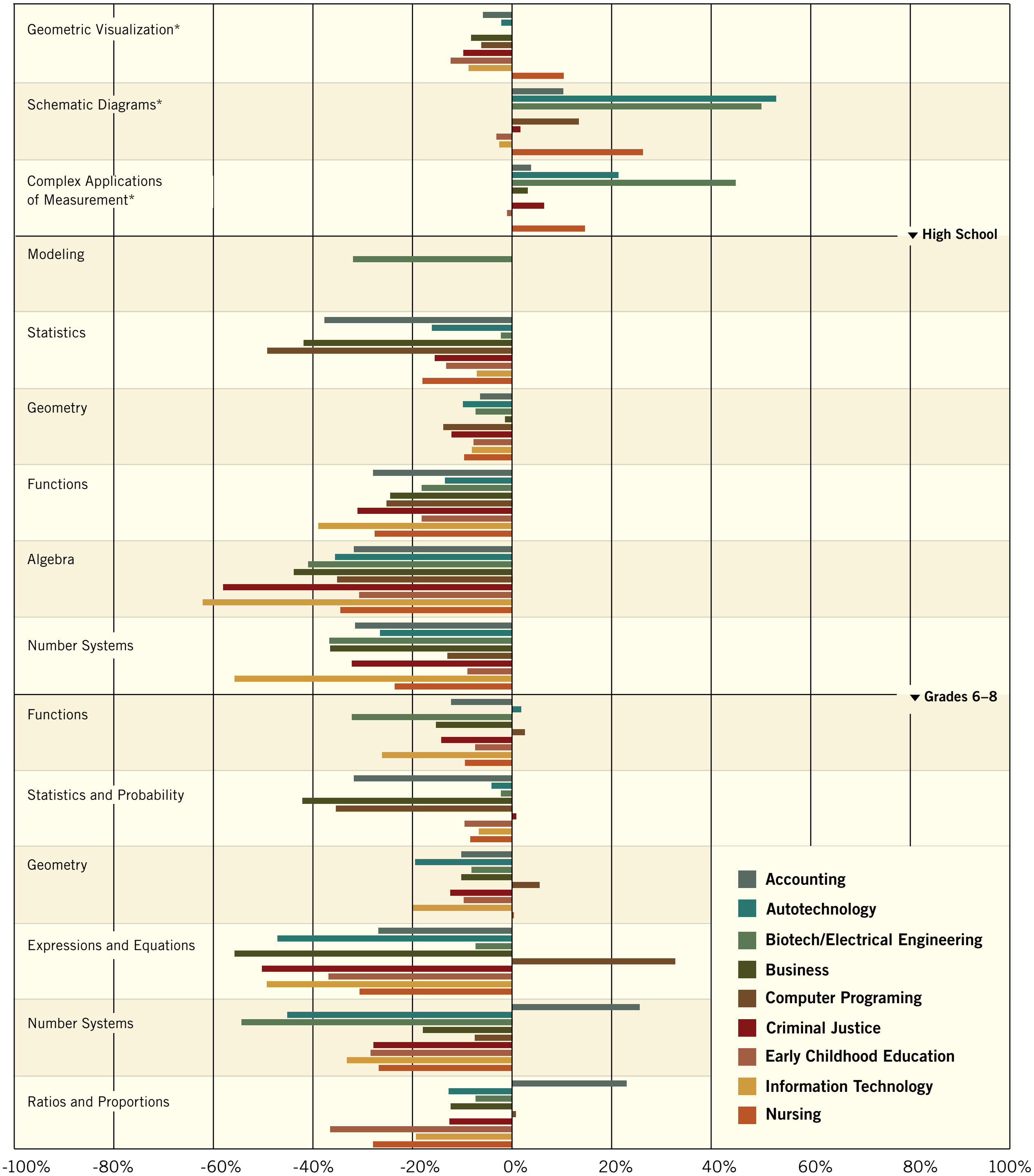


CHART 15 *Discrepancy Between Average Percent of Exam Items Containing Mathematics Content in 101 Courses and Average Percent of Exam Items Containing Mathematics Content in Required Mathematics Courses, All Programs*

XI. CONCLUSIONS

INITIAL REQUIRED COURSES in majors that most community college students take today (what we call 101 courses in this study) require very modest doses of mathematics and what they do require is mostly middle school mathematics along with some mathematics that is not typically taught in high school or in college. The mathematics that is used focuses on the application of middle school mathematics to career areas.

Only modest command of mathematics is required to do well on 101 course examinations, while the mathematics used in the 101 course textbooks is largely middle school mathematics. There is a heavy emphasis on proportionality (especially percent, rate and ratio but not solving "proportions") and, to a lesser extent, on mathematical domains not found in CCSSM: complex applications of measurement, schematic diagrams and geometric visualization.

The proportionality concepts often come up in measurement and data contexts that include explanatory text illustrated with charts, graphs and tables. Students do not have to perform algebraic manipulations or construct graphs or tables; but they do have to interpret the quantitative relationships in tables, graphs and formula to make full sense of the text.

One could compare the use of proportionality and statistics in the 101 texts to their use in newspapers and newscasts. For both texts and news, the reader plays a receptive role, comprehending but not producing the graphs, tables or calculations. In both contexts, the reader is using the mathematics to understand the quantitative aspects of a larger situation.

This prevalence of complex problems that draw on knowledge and skills first introduced in the middle grades was actually foreshadowed in CCSSM (p. 84):

> *The evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume. Because important standards for college and career readiness are distributed across grades and courses, systems for evaluating college and career readiness should reach as far back in the standards as Grades 6-8.*

Based on our findings, this statement is not only valid but crucial. It should be given importance in the implementation of the CCSSM, from textbooks to tests.

CCSSM's emphasis on modeling in high school can also open up a place in the curriculum for students to do the kind of work in mathematics that aligns to the actual community college needs articulated in this report. This suggests that the implementation of CCSSM shouldn't limit modeling to applications of advanced mathematics, but should also include sophisticated applications of knowledge and skills first learned in grades 6-8.[14]

The analysis of the textbooks uncovered the presence of three dimensions of mathematics that are not attended to by the CCSSM – the ability to interpret geometric visualizations, understand schematic diagrams and conduct complex applications of measurement of the sort periodically encountered in initial accounting, autotechnology, criminal justice, nursing and technology programs. It should also be noted that these aspects of mathematics are typically not found in the nation's high schools either.

The use of complex applications of measurement, schematic diagrams and geometric visualization not taught in high school may be more characteristic of how mathematics is typically applied on the job than in the classroom. In this respect, the 101 courses are doing a good job of shifting the focus to career preparation and away from academic abstractions.

Some courses had substantial spatial visualization and reasoning in the use of complicated 2-D schematics of 3-D objects and measurement. The geometry that was applied was not formal or deductive. Flow charts and decision trees played important roles in some texts, but these are not usually taught in school and are not addressed in the CCSSM. These three domains of non-standardized mathematics would be at home in a polytechnic curriculum such as those found in Northern Europe and East Asia and certainly are important for work in technological applications.

If the 101 courses do, indeed, reflect mathematics as it is applied on the job, it raises a disturbing question: are we turning away otherwise qualified youth from good jobs because they fail to meet irrelevant requirements? And for the students successful in reaching two-year programs, are the currently required high school mathematics courses wasting time and effort on the wrong mathematical priorities instead of spending it on more relevant mathematics? The 101 program courses demand different and much more modest mathematics than the mathematics found in the mathematics courses required for each program studied. Given the rate of failure in introductory community college mathematics courses, it appears that an artificial barrier to college success has been erected that ought to be pared back, while at the same time, strong consideration should be given to including the missing mathematics concepts and skills that are needed to succeed in initial credit bearing community college courses earlier in a student's school experience.

[14] For additional detail about this aspect of CCSSM, see http://commoncoretools.me/wp-content/uploads/2011/06/lastingachievementsink8.docx.

Required mathematics courses in community colleges generally review high school mathematics, and sometimes middle school mathematics. Still, the mathematics in the required mathematics courses often does not align well with the mathematics needed in the content courses. And, the mathematics that is found in both required mathematics courses and in their sponsoring program courses is typically not tested at the higher cognitive levels students require if they are to become fluent in applying their learning of mathematics to the fields of study in which they are majoring.

The most important preparation for the college mathematics courses observed in this study is solid understanding of middle school mathematics and some basic algebra as defined by the CCSSM. The implication is that, for students heading for the careers accessed through the community college courses we studied, K-12 should invest more time in this content even if it means less time in more advanced topics.

The most common initial required mathematics courses — college algebra, statistics and general mathematics — are quite redundant with respect to high school mathematics. If these mathematics topics were essential for work in the subject courses of the major, this might be a rational response to the lack of mathematics learning students bring from high school. It does not appear that this is the case, however. Community colleges should take a hard look at what is the most valuable mathematics for these programs. It may well be that high level use of middle school mathematics is most important. That is what the data here suggest.

One might reasonably ask whether it is fair to infer from our research on the requirements of the first year of community college the requirements of subsequent years. Put another way, one might ask, how do we know that more and more advanced mathematics is not required of students in their second and subsequent years of community college work for which they ought to be prepared in high school?

Our answer is as follows. It may well be the case that more mathematics and more advanced mathematics may be demanded of students in their second and subsequent years of community college than in the first year, but it is very unlikely that the teachers of those courses are assuming that students in those courses would have been prepared for that more advanced mathematics in their high schools. College catalogues in all kinds of institutions typically include language for 200 level courses that explicitly announces to students applying for entry into those courses that they need to take named 100 level courses as a prerequisite. Our assumption is that students who take the usual 100 level courses in mathematics or in programmatic courses with significant mathematics content and do well in them will be well prepared for the 200 level courses required by their program.

Mathematical proficiencies as defined by PISA are modest in the 101 courses and somewhat higher for initial required mathematics courses.

The expectations for mathematical proficiencies measured by PISA, such as problem solving, modeling and reasoning are modest in the 101 courses. Where they appear, they make only the most modest of cognitive demands with the exception of biotechnology and computer programming courses where more demanding skills are required.

The demands are higher across the board in the initial required mathematics courses and especially so for symbolism and formalism. Problem solving and representation are also moderately high. This, most likely, reflects standard mathematics teaching rather than the needs of students entering some of the most popular programs of study.

It should be noted that the analysis of the demand for PISA competencies in this study focuses on mathematics and steers clear of the comprehension demand of the surrounding non-mathematical text as this aspect of text complexity was not part of the rating process. So it would be a mistake to over interpret the PISA findings and conclude that higher order thinking is not in real demand for some 101 courses as it was hard not to notice some of it in passing. Other studies[15] have noted the importance of higher order, meta-cognitive and strategic thinking as well as character in predicting college success. This study does not contradict those findings but notes that such demands do not emanate from the use of mathematics in these courses.

The prerequisite mathematics needed to succeed in community college initial credit bearing courses are a good command of middle school mathematics and, in some cases, basic high school algebra. This raises questions about: 1) Algebra II requirements for high school graduation; 2) Placement test emphasis on geometry and advanced algebra to place into credit classes; and 3) What we teach in high school.

The analysis of the mathematics in the community college 101 introductory program and initial required mathematics courses suggests that the mathematical prerequisites for success are both more modest and somewhat different than what is generally called for in high school preparation. While little math is needed on the 101 introductory program course exams, a solid grasp of middle school mathematics and some high school algebra is needed to comprehend the texts in those classes and do well in the range of required mathematics courses that were analyzed.

This evidence does not support the notion that Algebra II is a necessary prerequisite to success in community college. Nor is the trend toward reducing time spent on middle school mathematics in favor of an early rush for students to take Algebra I in 8th grade. As a nation, it appears we are spending less time on the most important topics for success in the first year of community college, and more time on advanced mathematics that is used by a small fraction of the population when compared to those who need a command of middle school mathematics.

These results also raise questions about the relevance of much high school mathematics content, especially advanced algebra and geometry, on college placement examinations. Most of these topics are simply not found in community college introductory program courses, so competence in these topics is not needed in order to be adequately prepared for these community college programs. Placement examinations that demand that students master mathematics that they will not need to be successful in community college only serve to deny access to many students who are otherwise adequately prepared to succeed in the courses they will be required to take.

The lack of alignment between the initial required mathematics courses and the kind of applied mathematics used in the 101 courses raises questions about the design of community college programs, at least in terms of the mathematics requirements.

The analysis in this study calls into question the design of community college programs, at least with respect to the required mathematics needed for many careers. What mathematics is actually useful and important in these programs? This can be a matter

[15] Conley, D.T., Redefining College Readiness (2007). Eugene, OR: Education Policy Improvement Center.

of how useful it is in comprehending the subject as well as its usefulness in particular industries as one advances to positions of greater responsibility. It is time for serious research to take a fresh look at what mathematics is essential for students to build competence in fields and careers of promise, and what is not. The requirements for and content of mathematics courses should focus on what is important rather than what happens to have been inherited as policy.

One could argue that mastering college algebra is good discipline and good for an educated citizenry, but the first year of college is probably the last opportunity to teach mathematics to many students on the threshold of their careers. Certainly this opportunity should not be wasted on mathematics that happens to be on offer and likely of little direct benefit to most students when broadly based quantitative literacy courses might yield dividends for many years to come. For example, it seems the system is not taking good advantage of the opportunity to teach criminal justice technicians' the mathematics valuable to their work (e.g., statistics and data analysis) and hence valuable to everyone who depends on their work. Some fields, like nursing, have addressed this problem by developing special purpose mathematics courses, but every program shouldn't be backed into this corner.

Redundancy with high school mathematics is also problematic. It may be that this is a response to the realities of preparation students in these programs have or do not have. Nonetheless, we have to ask if this is the right mathematics to teach students who are enrolled in career-oriented programs with a course of study that requires mastery of a different kind of mathematics.

The picture that comes into focus from the evidence in this study is disturbing. We are leading our youth — starting in elementary school and continuing into middle school — through an irrational pathway of courses and requirements and the Common Core State Standards (CCSS) will not fix this problem. The upper levels of the CCSS in mathematics do not have high relevance for a student to be well prepared for success in community college 101 courses. While the Standards flag flies high, achievement data from NAEP and state testing show us that many students are not proficient in middle school mathematics, the very mathematics many of these same students need most for community college programs leading to good jobs. At the same time, these same students are rushed through middle school topics they aren't learning so they can take advanced mathematics courses of dubious relevance required for high school graduation. Some mathematics important in the 101 courses (and apparently in jobs) is neither in the CCSSM nor taught in high school mathematics. **In sum, much that is taught in high school is not needed, much that is taught in middle school is not learned, and some topics that are needed are neither taught nor learned.**

This is not just a matter of inefficiently allocating scarce and valuable resources. It is about sealing off opportunity and loading down with unnecessary debt young adults who deserve better—young adults who could succeed in college, have better lives and make more significant contributions to society.

This report will be jarring for many. Our findings paint a very different picture of the actual standards for success in our community colleges than many have been carrying around in their heads. While we are confident that our research techniques have enabled us to produce a much more accurate picture of those standards than the nation has ever had before, we do not regard this report as the last word on the subject. We would welcome studies that include a much larger random sample of colleges, take a closer look at colleges with outstanding reputations and gather a larger sample of the materials used in courses as well as student work. We think it would be worthwhile to do case studies of community colleges, looking in more detail at classroom practices and interviewing instructors to better understand why they are not making full use of the texts they assign and gauge their own sense of their students' needs and limitations. It is not unusual for researchers, in their reports, to call for more research, but we do believe that, in this case, more research could pay large dividends.

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APPENDICES

Philip Daro, *Co-Chair*

Philip Daro is a Senior Fellow for Mathematics for Pearson - America's Choice where he focuses on programs for students who are behind and algebra for all. He also directs the partnership of the University of California, Stanford and others with the San Francisco Unified School District for the Strategic Education Research Partnership, with a focus on mathematics and science learning among students learning English or developing academic English. Last year he chaired the Common Core State Standards Mathematics Workgroup.

Mr. Daro has directed, advised and consulted to a range of mathematics education projects. He currently serves on the NAEP Validity Studies panel, has chaired the mathematics standards committees for Georgia and Kentucky and chaired the Technical Advisory Group for ACHIEVE's Mathematics Work Group. He also has served on the College Board's Mathematics Framework Committee, the RAND Mathematics Education Study Panel and several mathematics task forces for the State of California. A regular consultant to large urban school districts across the country, from the mid '80s until the '90s, he was the director of the California Mathematics Project for the University of California. He has also worked with reading and literacy experts and panels on problems related to academic language development, especially in mathematics classroom discourse.

Solomon Garfunkel, *Co-Chair*

Solomon Garfunkel is the Executive Director of COMAP and has dedicated the last 35 years to research and development efforts in mathematics education. Dr. Garfunkel has been on the mathematics faculty of Cornell University and the University of Connecticut at Storrs. He has served as project director for the Undergraduate Mathematics and Its Applications Project (UMAP), the High School Mathematics and its Applications Project (HiMap), the Geometry and its Applications Project (GeoMap), the History of Mathematics and its Applications Project (HistoMap), the Technical Mathematics and its Applications Project (TechMap) funded by NSF, and directed three telecourse projects including *Against All Odds: Inside Statistics,* and *In Simplest Terms: College Algebra,* for the Annenberg/CPB Project.

Dr. Garfunkel has been the executive director of COMAP, located in Bedford, Massachusetts, since its inception in 1980. He was the project director and host for the series *For All Practical Purposes: Introduction to Contemporary Mathematics*, and co-principal investigator on the ARISE Project as well as the DynaMap Project. He was the Glenn Gilbert National Leadership Award Recipient for 2009 and currently serves on the Mathematics Expert Group of PISA.

Geri Anderson-Nielsen

Geri Anderson-Nielsen was a classroom teacher for more than thirty years. She has taught mathematics at almost every pre-college level: elementary, middle, and high school. For the past fifteen years she has concentrated on designing and delivering standards-based professional development workshops for pre-college and GED teachers around the country and visiting their schools to promote implementation of good mathematics teaching in the classroom. Her special area of interest has been in using computer and Internet technologies to support this work.

In 2003 she joined NCEE's writing team for *Preparatory Course for GED Mathematics* and after publication of the texts led workshops for GED teachers using these materials and followed up as a mentor for the teachers in their classrooms. Ms. Anderson-Nielsen has also been a member of national committees that support the teaching of strong mathematics in schools including the NAEP

2004 Mathematics Planning Committee; The National Board for Professional Teaching Standards Adolescence and Young Adulthood/Mathematics Committee (2000), and the Panel on College and University Programs of the Mathematical Sciences Education Board. She has also served on two of the NCTM Annual Meeting Program Committees, received the 1991 Presidential Award in Mathematics from the District of Columbia, and been an Einstein Congressional Fellow in the office of Senator Edward M. Kennedy.

John T. Baldwin

John T. Baldwin received his Ph.D. from Simon Fraser University in 1971. Since then he has published two books and many articles in *Mathematical Logic* (and a few in mathematics education) during a career at the University of Illinois at Chicago. Since the middle '80s he has been active in various aspects of mathematical education. He served from 1989-1993 on the Local School Council of Greene Elementary School. He was co-principal investigator in the early '90s of the College Preparatory Mathematics project, which significantly increased the number of students taking four years of mathematics at 12 high schools in Chicago and southeast Wisconsin. He was Assistant Head for Instruction in the Department of Mathematics, Statistics, and Computer Science and later the Director of Office of Mathematics Education at UIC. Dr. Baldwin retired from UIC in 2008 but continues to be active in both research and mathematics education. One of the leaders of the Chicago Algebra Initiative, he currently is a principal investigator (in math) for the NSF-funded Chicago Teacher Transformation Institute and is enjoying teaching a group of teachers a course on: Logic Across the High School Curriculum.

Patrick Callahan

Patrick Callahan is Co-Director of Special Projects at the California Mathematics Project at UCLA. He is a mathematician who has been actively involved in improving mathematics education, working with numerous projects at the state and national level. Dr. Callahan's research interests in mathematics education include the topology of teacher networks, specialized knowledge for teachers, and the cognitive development of children's geometric reasoning, while his research in mathematics has focused on low-dimensional hyperbolic geometry and topology. Previously, he was the Executive Director of the CalTeach Science and Mathematics Initiative at the University of California's Office of the President and a Mathematician in Residence for California's statewide Mathematics Professional Development Institutes. Before returning to California he helped develop and implement the UTeach Program at the University of Texas at Austin.

Andrew S.C. Chen

Andrew S.C. Chen is the President of EduTron Corporation located in Winchester, Massachusetts. Before founding EduTron he was a professor and a principal research scientist at the Massachusetts Institute of Technology. He frequently consults with education research institutions including the Institute for Education Science at the U.S. Department of Education and Achieve. Dr. Chen served on the Common Core State Standards Development Team in Mathematics. He is on the National Council on Teacher Quality's Advisory Board and the Massachusetts Board of Education's Mathematics and Science Advisory Council.

Dr. Chen provides professional development in mathematics and science to teachers at all levels through Intensive Immersion Institutes. He works with school districts and school administrators to increase their capacity to support excellent mathematics and science instruction. He also works with higher education institutions to develop rigorous and effective pre-service and in-service offerings in mathematics and science. Dr. Chen was an adviser for the Massachusetts 2008 Guidelines for the Mathematical Preparation of Elementary Teachers and continues to teach and do research in physics. He received a Ph.D. in physics from Columbia University.

Wade Ellis, Jr.

Wade Ellis, Jr. has taught mathematics at West Valley Community College in California for over 30 years. He earned degrees in mathematics from Oberlin College and The Ohio State University. He is a co-author of over 30 books on the learning and teaching of mathematics using technology and speaks regularly at regional, national and international conferences. Mr. Ellis is the recipient of the American Mathematics Association of Two-Year Colleges' Teaching Excellence Award and the MAA Northern California Section's Distinguished College or University Teacher Award recognizing his lifetime achievement. He is a former Second Vice-President of the Mathematical Association of America and a former member of The National Academies' Mathematical Sciences Education Board. For over 20 years he was a member of the Mathematics Diagnostic Testing Project of the University of California and has recently been a member of the Common Core State Standards Project's Mathematics Reviewing Committee. Mr. Ellis is currently a consultant to the Texas Instruments Educational Technology Company as a Senior Mathematics Advisor.

Robert L. Kimball, Jr.

Robert L. Kimball, Jr. taught at Wake Technical Community College in Raleigh, North Carolina from 1981 until 2011 when he retired. For 27 of those years, he was head of the Mathematics and Physics Department. Previously, he taught mathematics and coached in high school. Dr. Kimball was the founding president of NCMATYC and was a regional vice president of AMATYC. He has chaired the Technical Mathematics Committee of AMATYC and served on advisory boards for projects related to the workplace. In addition to writing a textbook and manuals, he was also a writer and consultant to AMATYC's *CROSSROADS in Mathematics: Standards for Introductory College Mathematics* as well as to *Beyond Crossroads*. He has been a project investigator on several ATE and CCLI awards from the National Science Foundation. He currently is engaged by the Carnegie Foundation for the Advancement of Teaching on its advisory committee for both *StatWay* and *MathWay* (now *QuantWay)* and is also serving on the team of authors writing lessons for *QuantWay*.

Lucy Hernandez Michal

Lucy Hernandez Michal is a mathematics professor at El Paso Community College (EPCC), having received a Bachelor of Science in Mathematics from the University of Texas at El Paso and a Master of Science in Mathematics at Michigan State University. She serves on EPCC's Student Success Core Team and is currently leading the President's Faculty Data and Research Team. Service to the college includes her participation in the Achieving the Dream's Core Strategy Team, chair of EPCC's Developmental Education Council, member of the El Paso College Readiness Consortium, and mathematics coordinator for EPCC's Rio Grande Campus. She is also a member of EPCC's *Statway* Team as part of a national effort convened by the Carnegie Foundation for the Advancement of Teaching.

She has worked on aligning mathematics curriculum, instruction and assessment at the national, state and local levels. Locally, at the El Paso Collaborative for Academic Excellence, she was director of the K-16 Mathematics Alignment Initiative funded by the Pew Charitable Trusts and Director of Mathematics and Science for the Mathematics and Science Partnership funded by NSF. At the state level, she served on the Texas State Mathematics Vertical Team convened by the Texas Higher Education Board and the Texas Education Agency to draft the State's College Readiness Standards in mathematics. Nationally, she served on a Mathematics Advisory Panel (MAP) for Achieve. She also served on a panel convened by the American Institutes for Research to review items for the National Assessment of Educational Progress (NAEP) and did a comparison of the NAEP mathematics standards against the state standards of six other states including Texas.

Lisa Seidman

Lisa Seidman is a professor and Biotechnology Program Director (on a rotating basis) at Madison Area Technical College (MATC) in Wisconsin. She received a Ph.D. in biology from the University of Wisconsin and worked as a post-doctoral fellow in the Medical School at Yale University and the Department of Human Oncology at the University of Wisconsin before becoming an instructor in the Biotechnology Laboratory Technician Program at MATC in 1987. Since then she has developed a math program for biotechnology students that was selected as an exemplary program in 2001 by AMATYC, been the PI or Co-PI for four grants from the National Science Foundation and was selected in a national competition to be a faculty mentor by the American Association of Community Colleges and NSF in 2008-2010.

Dr. Seidman is presently co-PI of Bio-Link, a national consortium of biotechnology educational programs that is headed by the City College of San Francisco. She is the lead author of three textbooks for biotechnology students: *Basic Laboratory Methods for Biotechnology: Textbook and Laboratory Reference; Basic Laboratory Calculations for Biotechnology;* and *Laboratory Manual for Biotechnology and Laboratory Science: The Basics*. Most recently she has been instrumental in developing a stem cell technologies certificate program at MATC.

Colin L. Starr

Colin L. Starr is an associate professor and chair of mathematics at Willamette University in Oregon. He was an undergraduate double major in math and physics at Linfield College and earned his MS and Ph.D. in Mathematics from the University of Kentucky in 1995 and 1998, respectively. He is a co-PI on the Willamette Valley Mathematics Consortium, a REU-RET joining four Willamette Valley colleges. He loves working with students and, in addition to several mathematics research projects with undergraduates, serves as the advisor to the Oregon Zeta chapter of PME, the Willamette University Math Club, the Putnam Exam team, and the Willamette University Racquetball Club. His research interests are in algebra, graph theory and matroid theory, and he has served as a "Content Expert" for various K-12 educational entities including Harcourt Educational Measurement (as an item writer/reviewer for state standardized tests), Measured Progress (as an item reviewer for state standardized tests), and EPIC (as a writer/reviewer/consultant for projects concerning state standards and the Common Core).

I. Grades K–5

K–5: CC. COUNTING AND CARDINALITY

1. Know number names and the count sequence.
2. Count to tell the number of objects.
3. Compare numbers.

K–5: OA. OPERATIONS AND ALGEBRAIC THINKING

1. Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.
2. Represent and solve problems involving addition and subtraction.
3. Understand and apply properties of operations and the relationship between addition and subtraction.
4. Add and subtract within 20.
5. Work with addition and subtraction equations.
6. Work with equal groups of objects to gain foundations for multiplication.
7. Represent and solve problems involving multiplication and division.
8. Understand properties of multiplication and the relationship between multiplication and division.
9. Multiply and divide within 100.
10. Solve problems involving the four operations and identity and explain patterns in arithmetic.
11. Use the four operations with whole numbers to solve problems.
12. Gain familiarity with factors and multiples.
13. Generate and analyze patterns.
14. Write and interpret numerical expressions.
15. Analyze patterns and relationships.

K–5: NBT. NUMBER AND OPERATIONS IN BASE TEN

1. Work with numbers 11–19 to gain foundations for place value.
2. Extend the counting sequence.
3. Understand place value.
4. Use place value understanding and properties of operations to add and subtract.
5. Use place value understanding and properties of operations to perform multi-digit arithmetic.
6. Generalize place value understanding for multi-digit whole numbers.
7. Understand the place value system.
8. Perform operations with multi-digit whole numbers and with decimals to hundredths.

K–5: NOF. NUMBER AND OPERATIONS—FRACTIONS

1. Develop understanding of fractions as numbers.
2. Extend understanding of fraction equivalence and ordering.
3. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4. Understand decimal notation for fractions, and compare decimal fractions.
5. Use equivalent fractions as a strategy to add and subtract fractions.
6. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

K–5: MD. MEASUREMENT AND DATA

1. Describe and compare measurable attributes.
2. Classify objects and count the number of objects in categories.
3. Measure lengths indirectly and by iterating length units.
4. Tell and write time.
5. Represent and interpret data.
6. Measure and estimate lengths in standard units.
7. Relate addition and subtraction to length.
8. Work with time and money.
9. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
10. Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
11. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
12. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
13. Geometric measurement: understand concepts of angle and measure angles.
14. Convert like measurement units within a given measurement system.
15. Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

K–5: G. GEOMETRY

1. Identify and describe shapes.
2. Analyze, compare, create, and compose shapes.
3. Reason with shapes and their attributes.
4. Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
5. Graph points on the coordinate plane to solve real-world and mathematical problems.
6. Classify two-dimensional figures into categories based on their properties.

II. Grades 6–8

6–8: RP RATIOS AND PROPORTIONAL RELATIONSHIPS

1. Understand ratio concepts and use ratio reasoning to solve problems.
2. Analyze proportional relationships and use them to solve real-world and mathematical problems.

6–8: NS. THE NUMBER SYSTEM

1. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
2. Compute fluently with multi-digit numbers and find common factors and multiples.
3. Apply and extend previous understandings of numbers to the system of rational numbers.
4. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
5. Know that there are numbers that are not rational, and approximate them by rational numbers.

6–8: EE. EXPRESSIONS AND EQUATIONS

1. Apply and extend previous understandings of arithmetic and algebraic expressions.
2. Reason about and solve one-variable equations and inequalities.
3. Represent and analyze quantitative relationships between dependent and independent variables.
4. Use properties of operations to generate equivalent expressions.
5. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
6. Work with radicals and integer exponents.
7. Understand the connections between proportional relationships, lines, and linear equations.
8. Analyze and solve linear equations and pairs of simultaneous linear equations.

6–8: G. GEOMETRY

1. Solve real-world and mathematical problems involving area, surface area, and volume.
2. Draw, construct and describe geometrical figures and describe the relationship between them.
3. Understand congruence and similarity using physical models, transparencies, or geometry software.
4. Understand and apply the Pythagorean Theorem.
5. Solve real-life and mathematical problems involving volume of cylinders, cones, and spheres.
6. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

6–8: SP. STATISTICS AND PROBABILITY

1. Develop understanding of statistical variability.
2. Summarize and describe distributions.
3. Use random sampling to draw inferences about a population.
4. Draw informal comparative inferences about two populations.
5. Investigate chance process and develop, use, and evaluate probability models.
6. Investigate patterns of association in bi-variate data.

6–8: F. FUNCTIONS

1. Define, evaluate, and compare functions.
2. Use functions to model relationships between quantities.

III. High School (Grades 9–12)

High School: Number and Quantity

N-RN. THE REAL NUMBER SYSTEM

1. Extend the properties of exponents to rational exponents.
2. Use properties of rational and irrational numbers.

N-Q. QUANTITIES

1. Reason quantitatively and use units to solve problems.

N-CN. THE COMPLEX NUMBER SYSTEM

1. Perform arithmetic operations with complex numbers.
2. Represent complex numbers and their operations on the complex plane.
3. Use complex numbers in polynomial identities and equations.

N-VM. VECTOR AND MATRIX QUANTITIES

1. Represent and model with vector quantities.
2. Perform operations on vectors.
3. Perform operations on matrices and use matrices in applications.

High School: Algebra

A-SSE. SEEING STRUCTURE IN EXPRESSIONS

1. Interpret the structure of expressions.
2. Write expressions in equivalent forms to solve problems.

A-APR. ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS

1. Perform arithmetic operations on polynomials.
2. Understand the relationship between zeros and factors of polynomials.
3. Use polynomial identities to solve problems.
4. Rewrite rational expressions.

A-CED. CREATING EQUATIONS

1. Create equations that describe numbers or relationships.

A-REI. REASONING WITH EQUATIONS AND INEQUALITIES

1. Understand solving equations as a process of reasoning and explain the reasoning.
2. Solve equations and inequalities in one variable.
3. Solve systems of equations.
4. Represent and solve equations and inequalities graphically.

High School: Functions

F-IF. INTERPRET FUNCTIONS

1. Understand the concept of a function and use function notation.
2. Interpret functions that arise in applications in terms of the context.
3. Analyze functions using different representations.

F-BF. BUILDING FUNCTIONS

1. Build a function that models a relationship between two quantities.
2. Build new functions from existing functions.

F-LE. LINEAR, QUADRATIC, AND EXPONENTIAL MODELS

1. Construct and compare linear, quadratic, and exponential models and solve problems.
2. Interpret expressions for functions in terms of the situation they model.

F-TF. TRIGONOMETRIC FUNCTIONS

1. Extend the domain of trigonometric functions using the unit circle.
2. Model periodic phenomena with trigonometric functions.
3. Prove and apply trigonometric identities.

High School: Geometry

G-CO. CONGRUENCE

1. Experiment with transformations in the plane.
2. Understand congruence in terms of rigid motions.
3. Prove geometric theorems.
4. Make geometric constructions.

G-SRT. SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY

1. Understand similarity in terms of similarity transformations.
2. Prove theorems involving similarity.
3. Define trigonometric ratio and solve problems involving right triangles.
4. Apply trigonometry to general triangles.

G-C. CIRCLES

1. Understand and apply theorems about circles.
2. Find arc lengths and areas of sectors of circles.

G-GPE. EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS

1. Translate between the geometric description and the equation for a conic section.
2. Use coordinates to prove simple geometric theorems algebraically.

G-GMD. GEOMETRIC MEASUREMENT AND DIMENSION

1. Explain volume formulas and use them to solve problems.
2. Visualize relationships between two-dimensional and three-dimensional objects.

G-MG. MODELING WITH GEOMETRY

1. Apply geometric concepts in modeling situations.

High School: Statistics and Probability

S-ID. INTERPRETING CATEGORICAL AND QUANTITATIVE DATA

1. Summarize, represent, and interpret data on a single count or measurement variable.
2. Summarize, represent, and interpret data on two categorical and quantitative variables.
3. Interpret linear models.

S-IC. MAKING INFERENCES AND JUSTIFYING CONCLUSIONS

1. Understand and evaluate random processes underlying statistical experiments.
2. Make inferences and justify conclusions from sample surveys, experiments and observational studies.

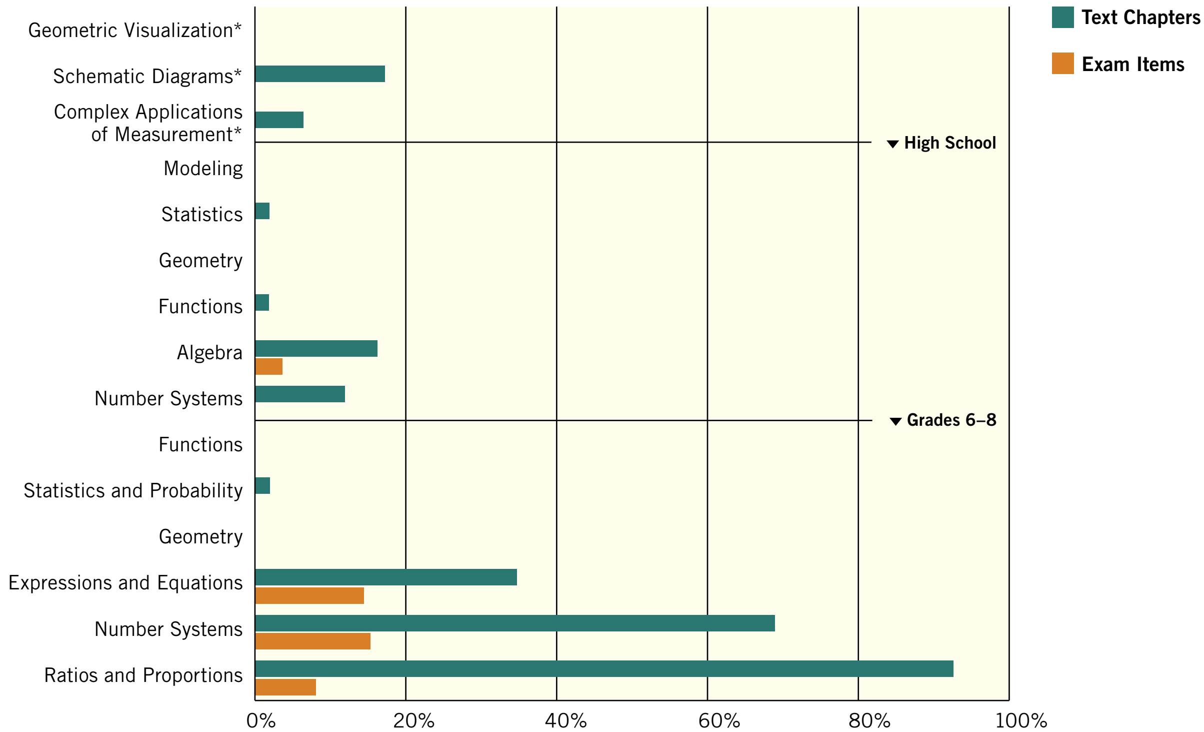
S-CP. CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY

1. Understand independence and conditional probability and use them to interpret data.
2. Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S-MD. USING PROBABILITY TO MAKE DECISIONS

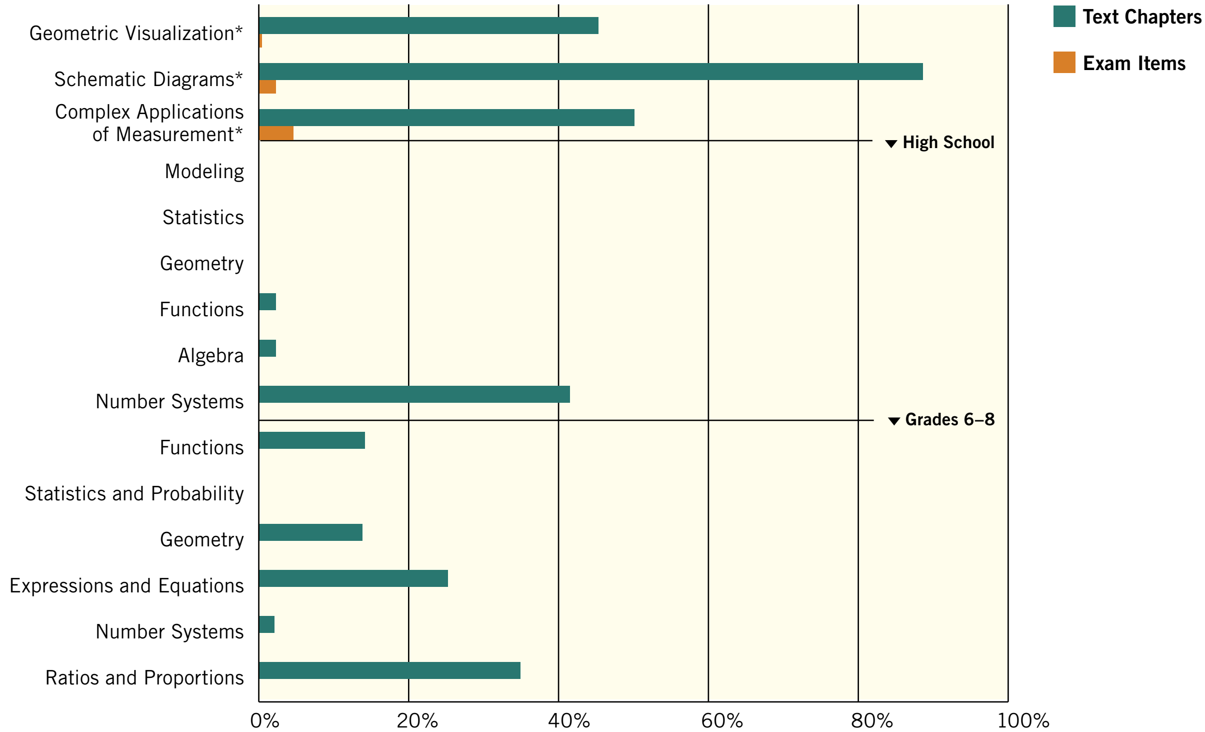
1. Calculate expected values and use them to solve problems.
2. Use probability to evaluate outcomes of decisions.

CHART C1 *Average Percent of Text Chapters and Exam Items Containing CCSSM Domains for Accounting Courses*



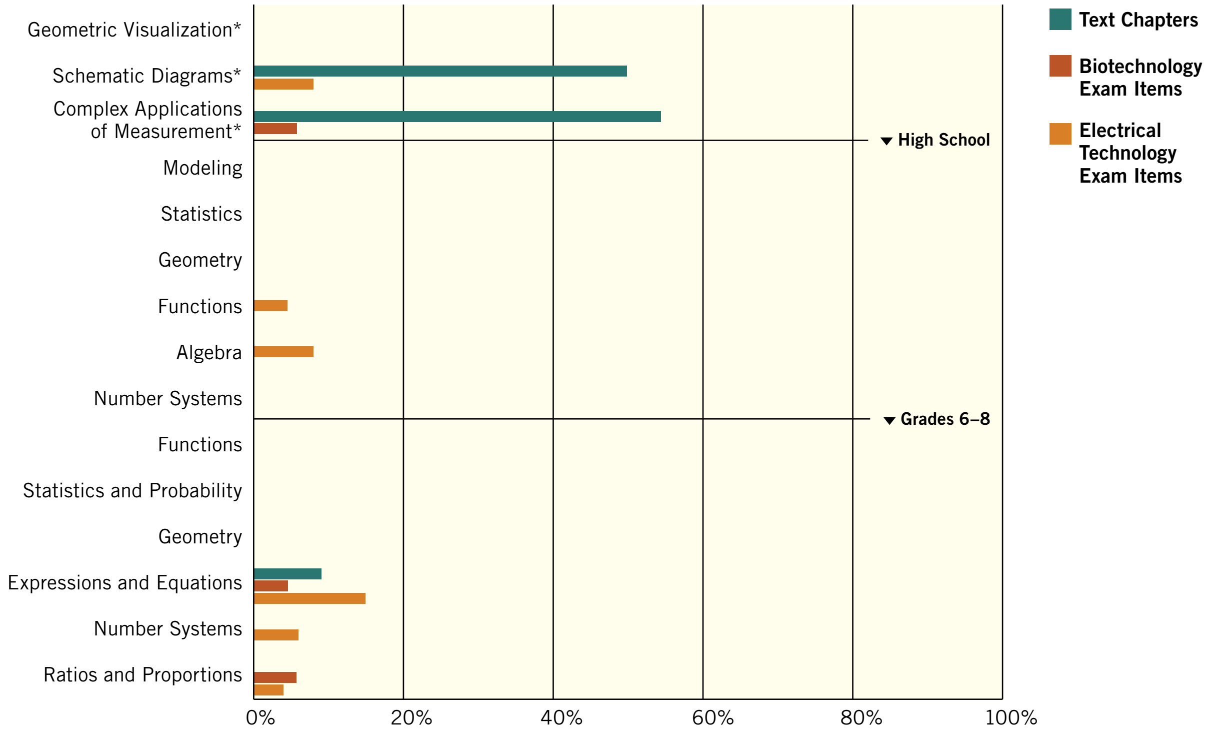
**Non-CCSSM domains added by the panel.*

CHART C2 *Average Percent of Text Chapters and Exam Items Containing CCSSM Domains for Autotechnology Courses*



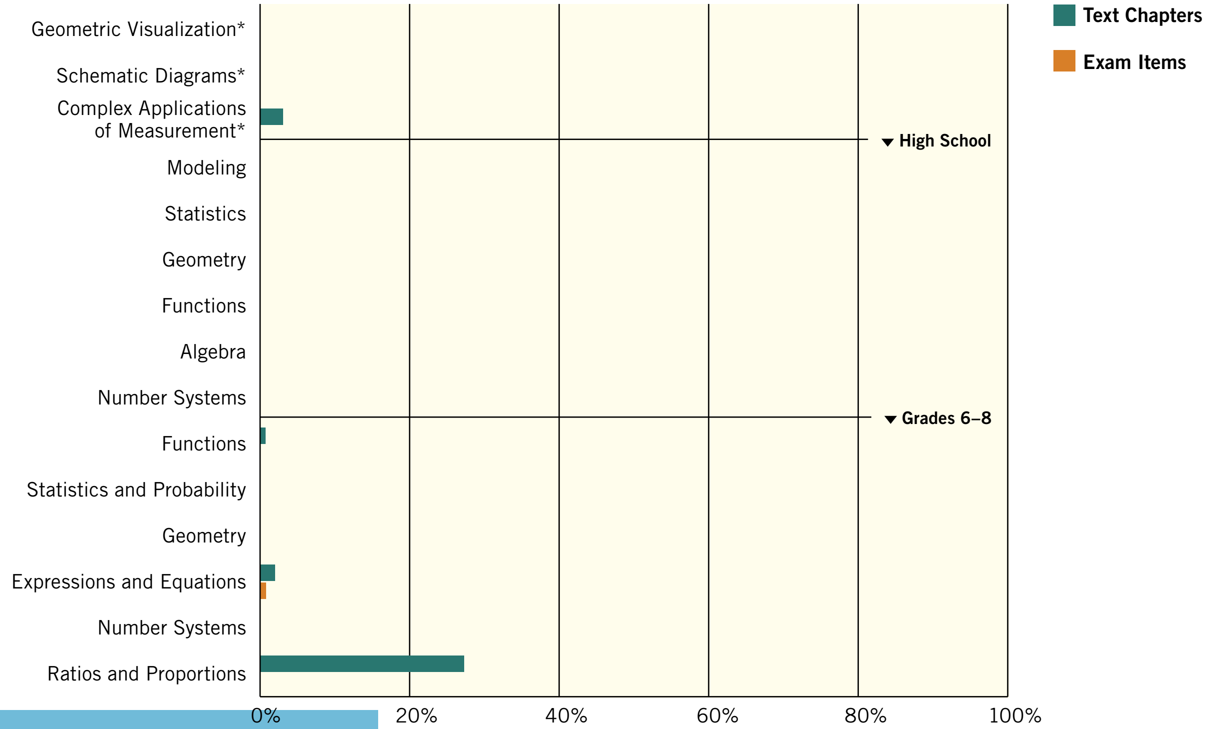
NB: Since there are no Early Childhood Education courses which contain mathematics, Early Childhood Education is not represented among these charts.
**Non-CCSSM domains added by the panel.*

CHART C3 *Average Percent of Text Chapters and Exam Items Containing CCSSM Domains for Biotech/Electrical Technology Courses*



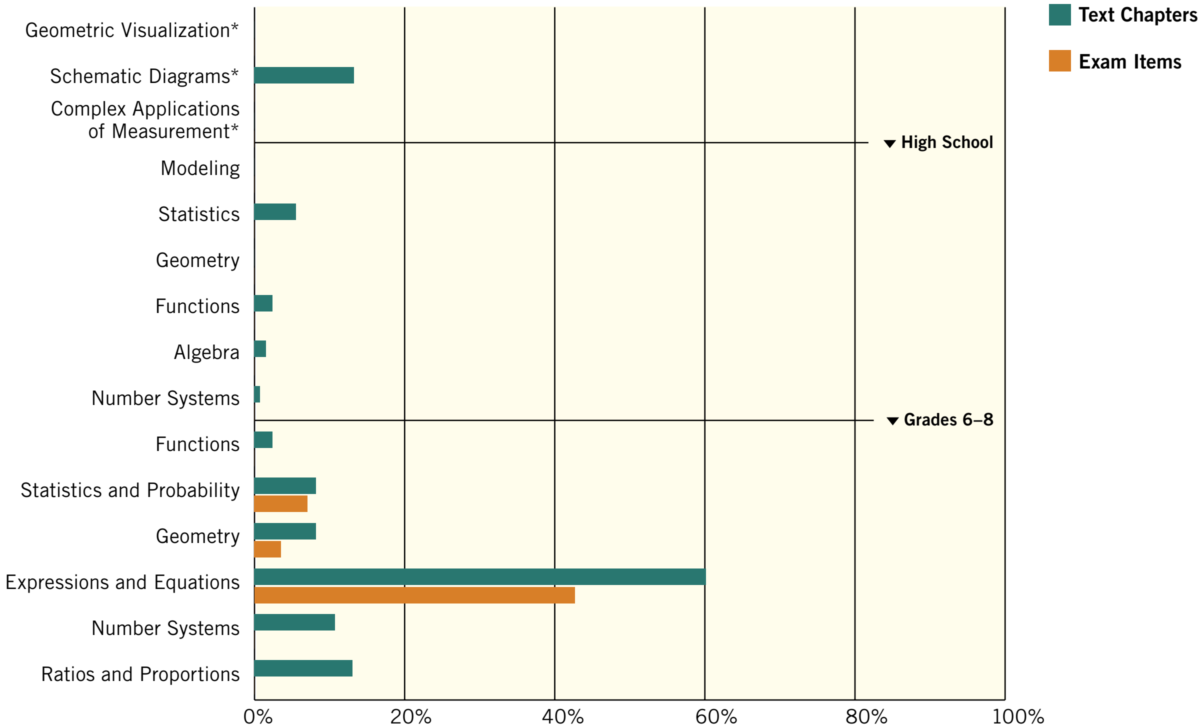
*Non-CCSSM domains added by the panel.

CHART C4 *Average Percent of Text Chapters and Exam Items Containing CCSSM Domains for Business Courses*



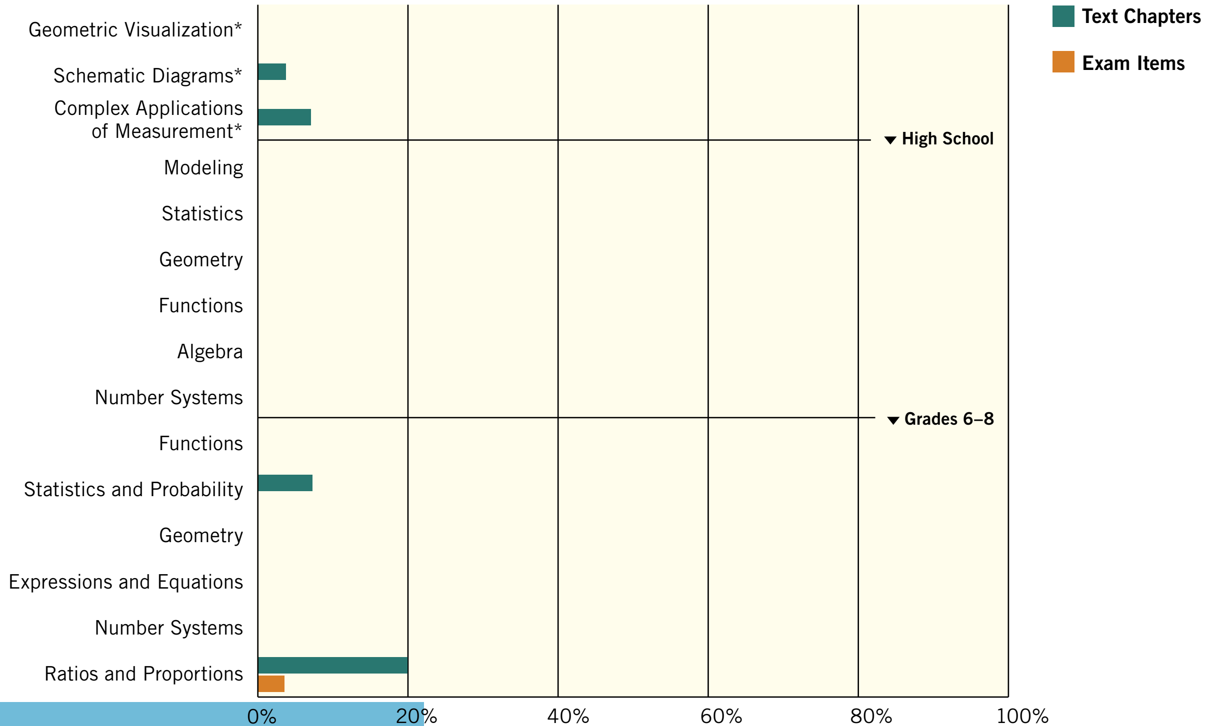
*Non-CCSSM domains added by the panel.

CHART C5 *Average Percent of Text Chapters and Exam Items Containing CCSSM Domains for Computer Programing Courses*



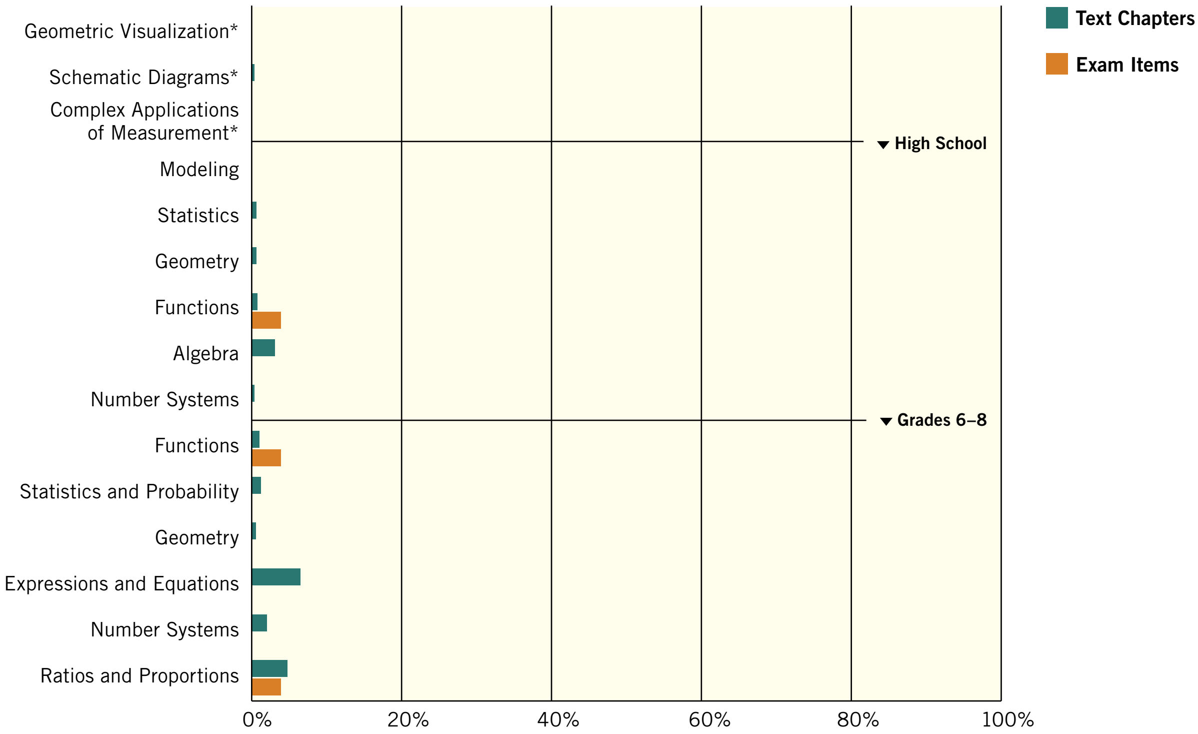
*Non-CCSSM domains added by the panel.

CHART C6 *Average Percent of Text Chapters and Exam Items Containing CCSSM Domains for Criminal Justice Courses*



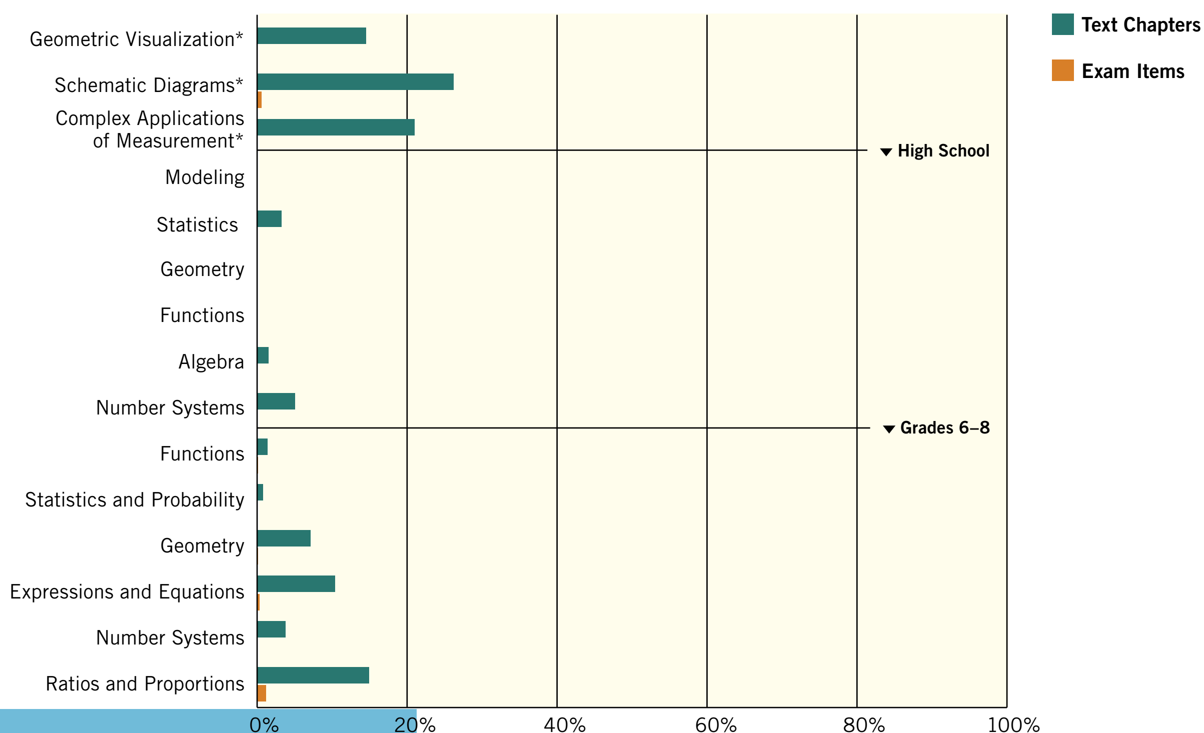
*Non-CCSSM domains added by the panel.

CHART C7 *Average Percent of Text Chapters and Exam Items Containing CCSSM Domains for Information Technology Courses*



**Non-CCSSM domains added by the panel.*

CHART C8 *Average Percent of Text Chapters and Exam Items Containing CCSSM Domains for Nursing Courses*



**Non-CCSSM domains added by the panel.*

CHART D1 *Average Percent of Text Chapters and Exam Items Rated at Each PISA Level for Accounting Courses*

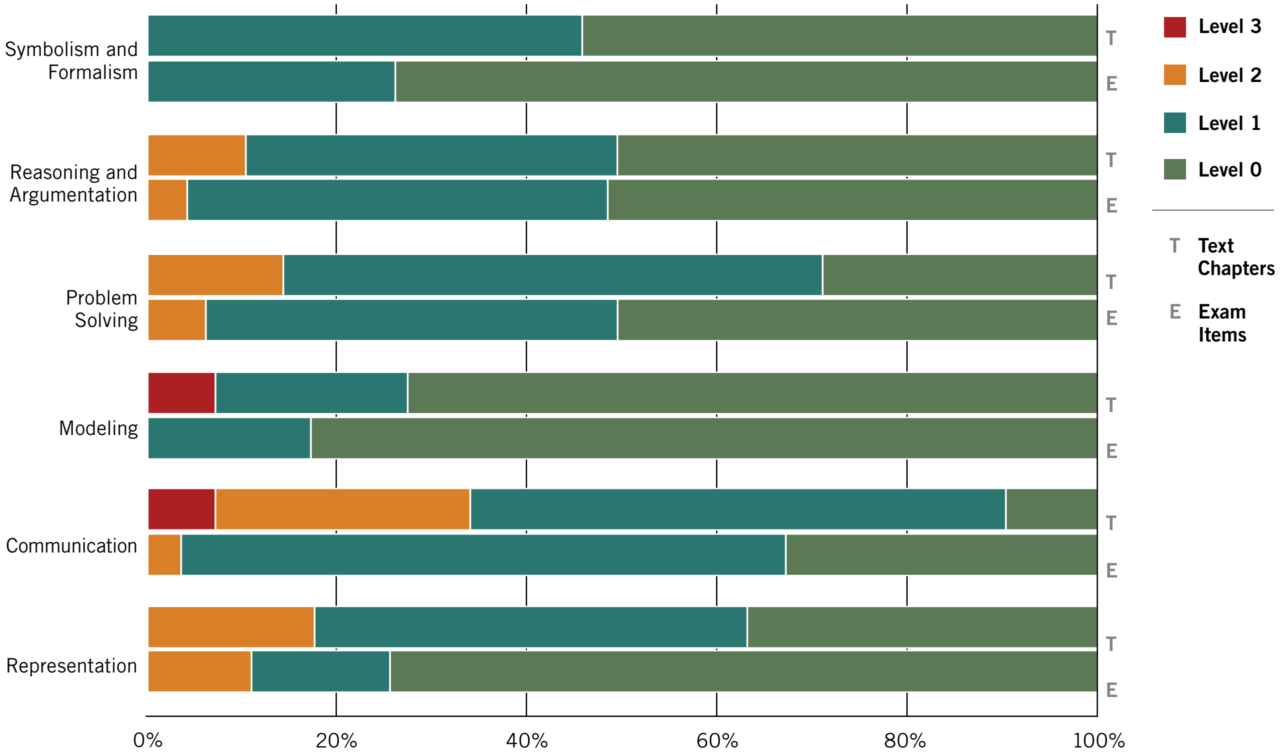
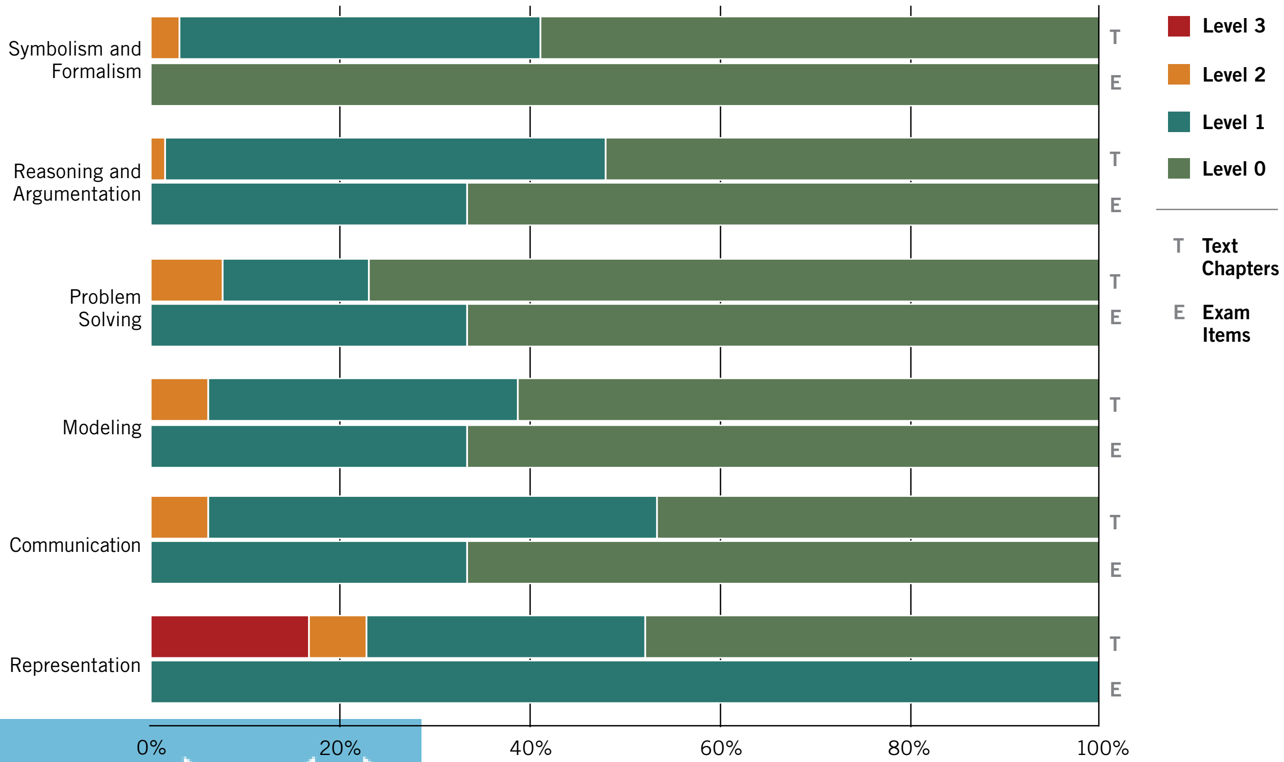


CHART D2 *Average Percent of Text Chapters and Exam Items Rated at Each PISA Level for Autotechnology Courses*



NB: Since there are no Early Childhood Education courses which contain mathematics, Early Childhood Education is not represented among these charts.

CHART D3 *Average Percent of Text Chapters and Exam Items Rated at Each PISA Level for Biotech/Electrical Technology Courses*

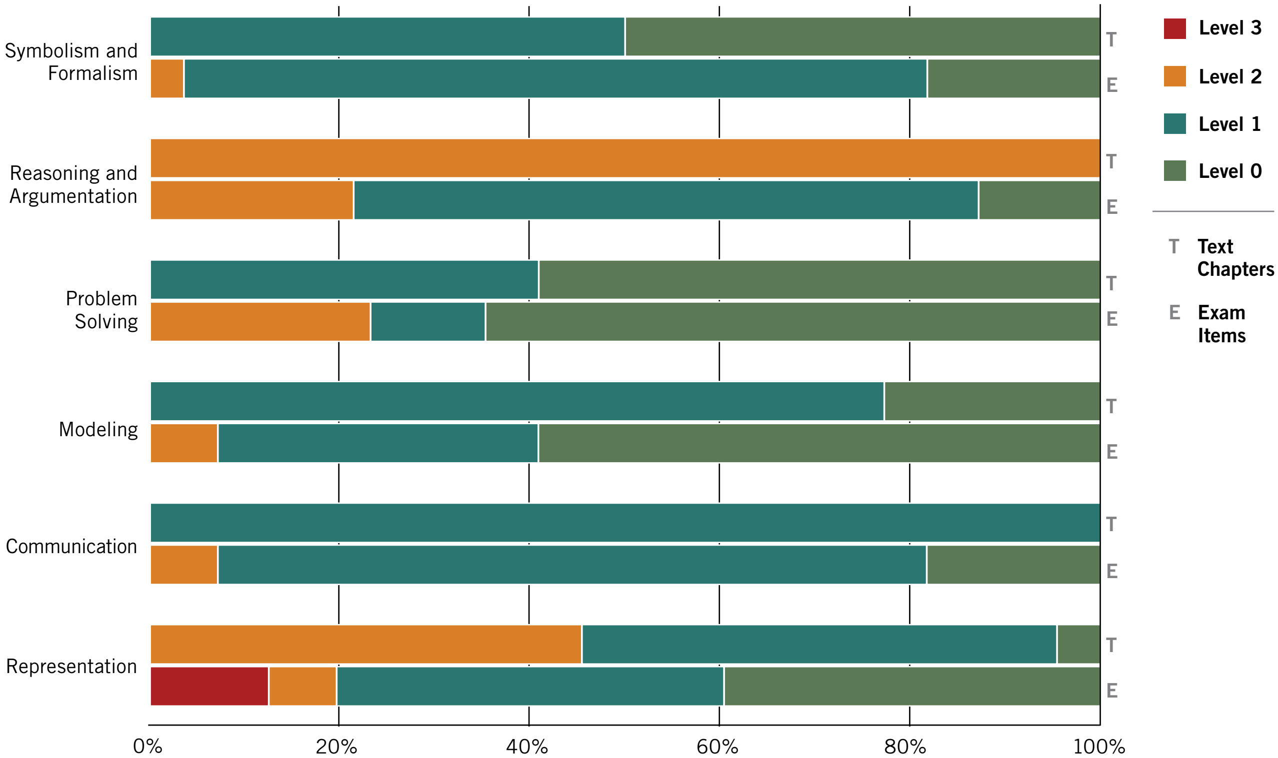


CHART D4 *Average Percent of Text Chapters and Exam Items Rated at Each PISA Level for Business Courses*

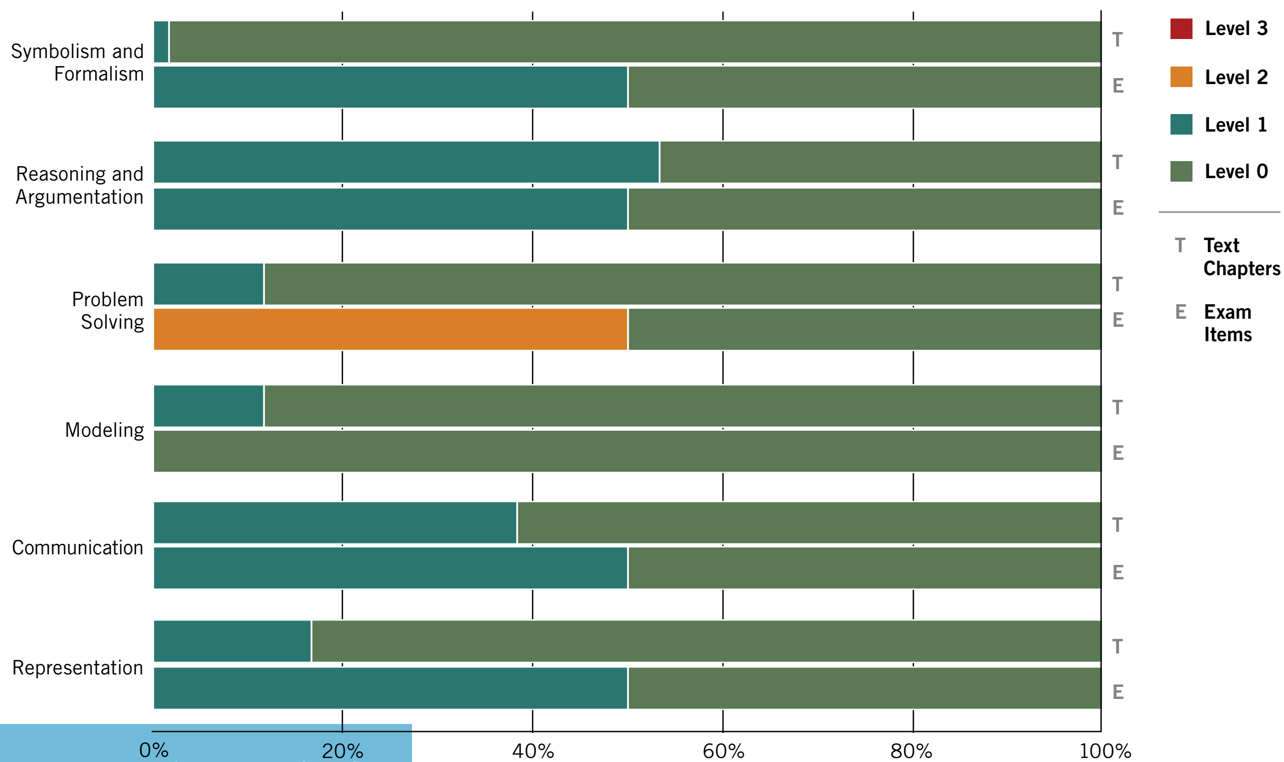


CHART D5 *Average Percent of Text Chapters and Exam Items Rated at Each PISA Level for Computer Programing Courses*

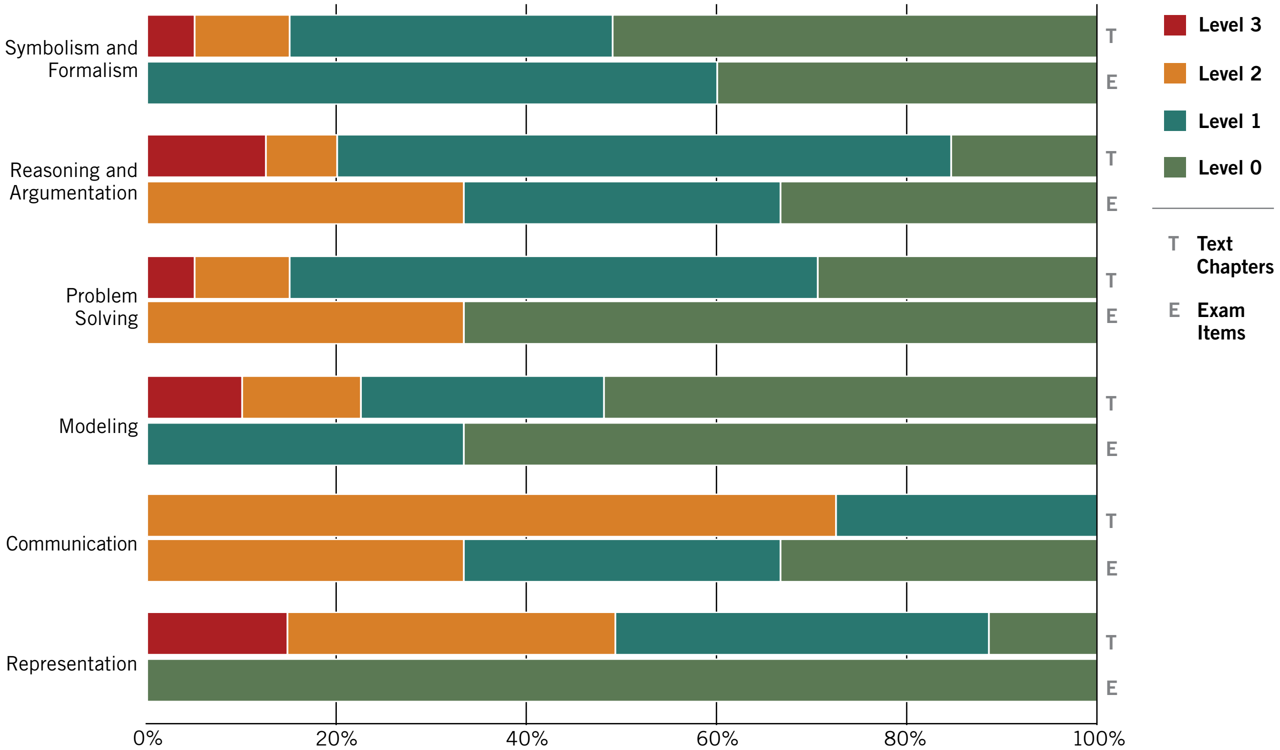


CHART D6 *Average Percent of Text Chapters and Exam Items Rated at Each PISA Level for Criminal Justice Courses*

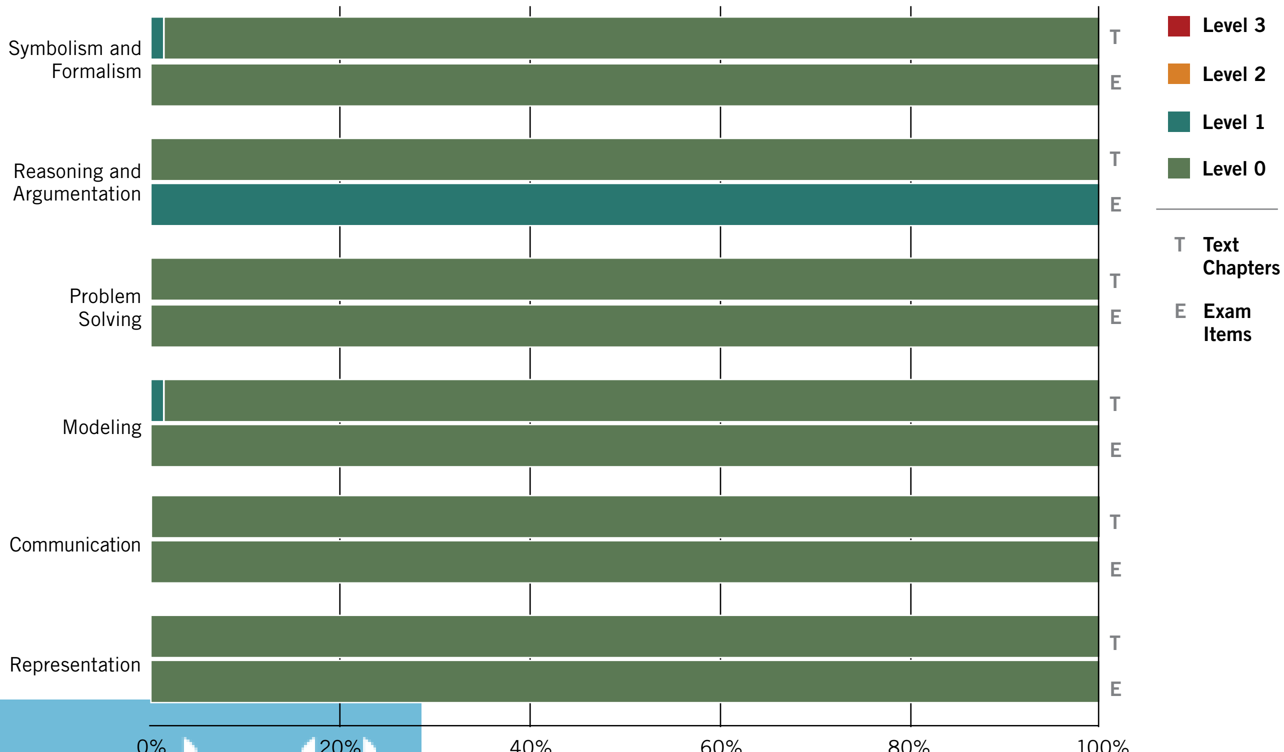


CHART D7 *Average Percent of Text Chapters and Exam Items Rated at Each PISA Level for Information Technology Courses*

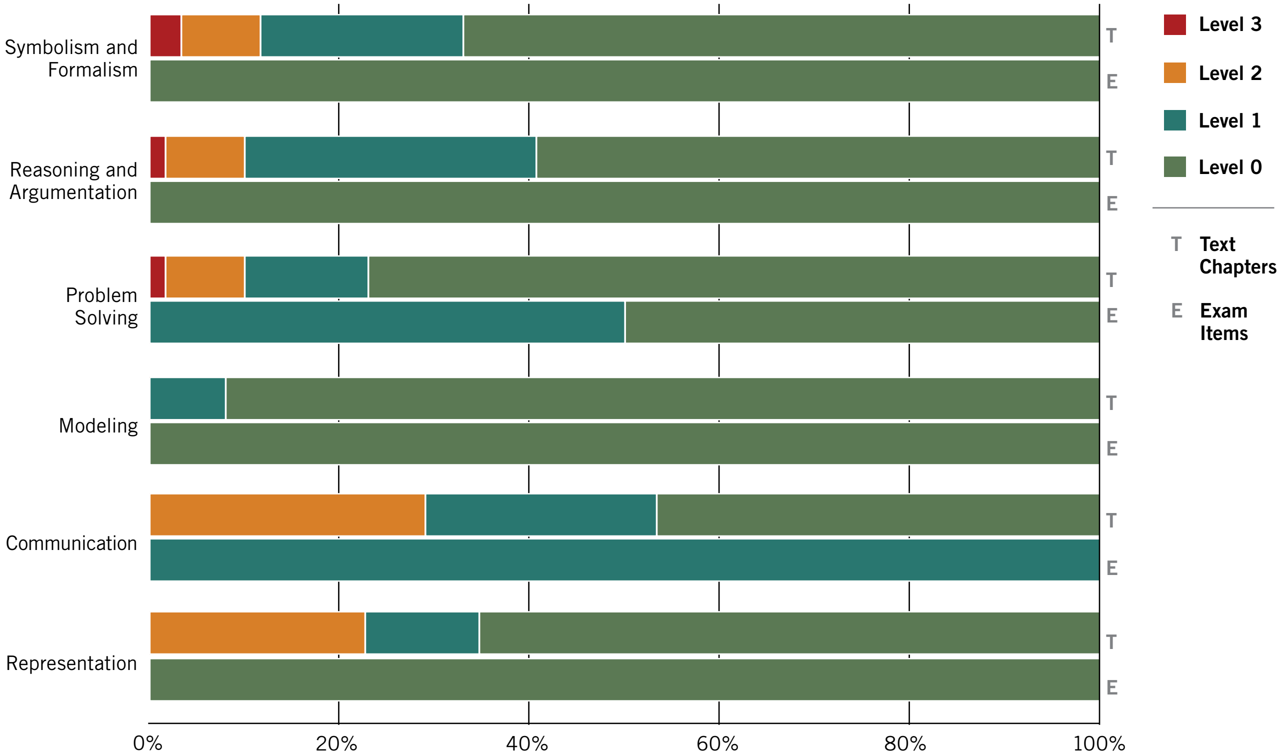
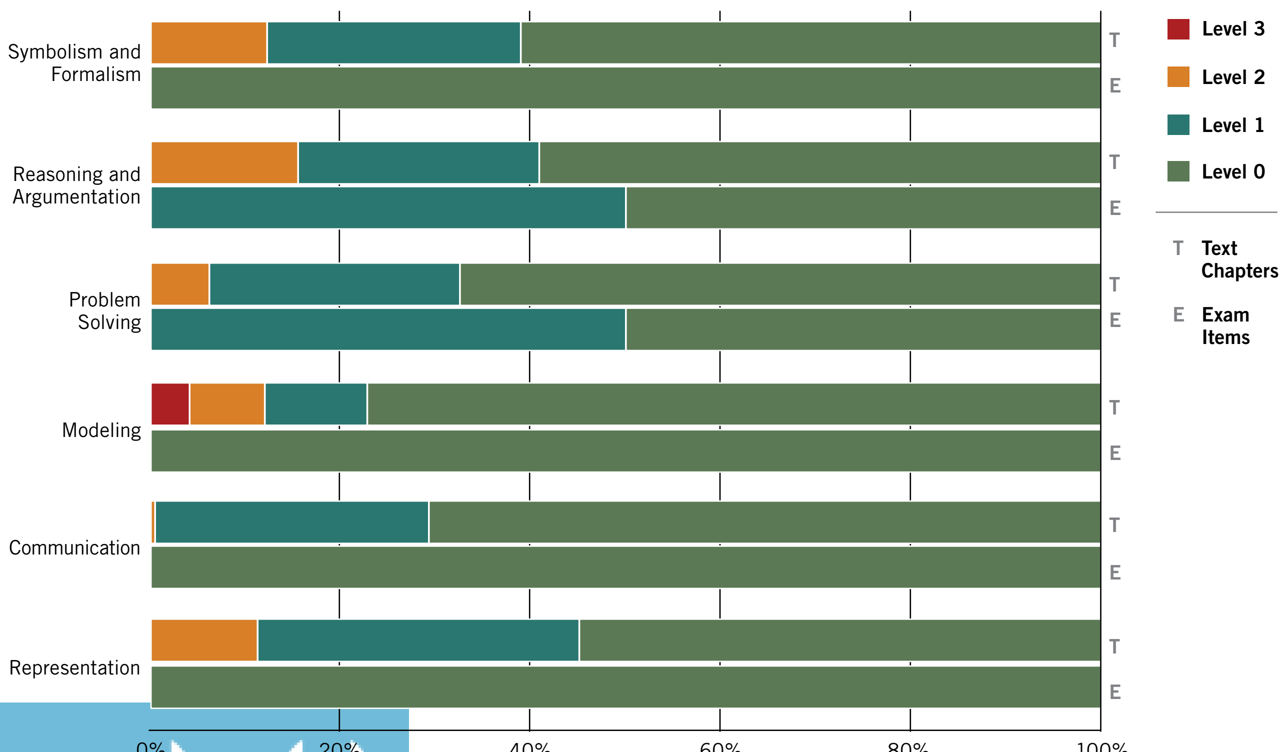


CHART D8 *Average Percent of Text Chapters and Exam Items Rated at Each PISA Level for Nursing Courses*



ACCOUNTING

101 Course Title	Initial Required Mathematics Course
Accounting 1	Business Math
Fundamental Accounting 1	Business Math or Applied Math
Introduction to Accounting and Financial Reporting I	Applied College Math
Principals of Accounting I	Elementary Statistics 1
Accounting	College Algebra
Financial Accounting	Principals of Statistics or College Algebra
Accounting 1	Intermediate Algebra*

AUTOTECHNOLOGY

101 Course Title	Initial Required Mathematics Course
Introduction to Automotive Systems	Intermediate Algebra
Automotive Emission Systems I	Applied Math**
Integrated Automotive Systems	Applications for Business and Other Careers or Topics in Contemporary Mathematics

BIOTECH/ELECTRICAL TECHNOLOGY

101 Course Title	Initial Required Mathematics Course
Introduction to Biotechnology	Math for Biotechnology or Introduction to Probability and Statistics*
Circuits 1	College Algebra
Introduction to Biotechnology	College Algebra

Each box represents a different course at a different community college
**Course not included in analysis because materials were not available*
***Students have a choice of which mathematics course to take. The lowest level course among the options was selected for the analysis.*
****Not included in mathematics analysis, as content not covered by CCSSM*

BUSINESS

101 Course Title	Initial Required Mathematics Course
Introduction to Business	Business Math*
Introduction to Business	Business Math or Applied Math
Introduction to Business	Elementary Statistics 1
Introduction to Business	Applications for Business and Other Careers

COMPUTER PROGRAMMING

101 Course Title	Initial Required Mathematics Course
Programming with C++	Business Calculus***
Introduction to Visual Programming	Elementary Statistics 1

CRIMINAL JUSTICE

101 Course Title	Initial Required Mathematics Course
Introduction to Criminal Justice	Intermediate Algebra
Introduction to Criminal Justice	Applied Math**
Criminal Procedures	Applied College Math
Introduction to Criminal Justice	College Algebra
Introduction to Criminal Justice	College Algebra
Introduction to Criminal Justice	Topics in Contemporary Mathematics or higher
Introduction to Criminal Justice	Math Concepts/Applications

EARLY CHILDHOOD EDUCATION

101 Course Title	Initial Required Mathematics Course
Childhood Growth Development and Learning	Math for Elementary School Teachers
Introduction to Early Childhood Education	Applied Math**
Early Childhood Growth and Development	Applied College Math**
Foundations of American Education	Math for Elementary School Teachers
Human Growth and Development	College Algebra
Introduction to Early Childhood Education	Topics in Contemporary Mathematics
Survey of Early Childhood Education	Math Concepts/Applications

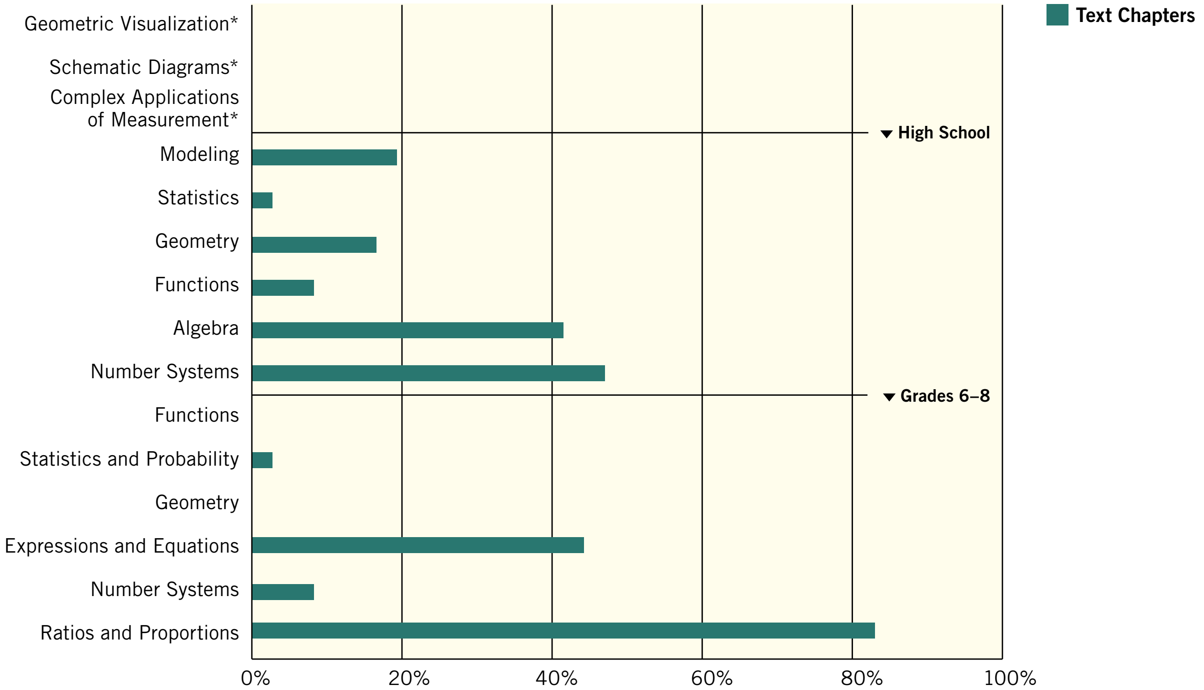
INFORMATION TECHNOLOGY

101 Course Title	Initial Required Mathematics Course
Introduction to Computers	College Algebra
Introduction to Computers	Intermediate Algebra
Computer Information Systems	Intermediate Algebra
Introduction to Computers	College Algebra
Introduction to Computers	College Algebra

NURSING

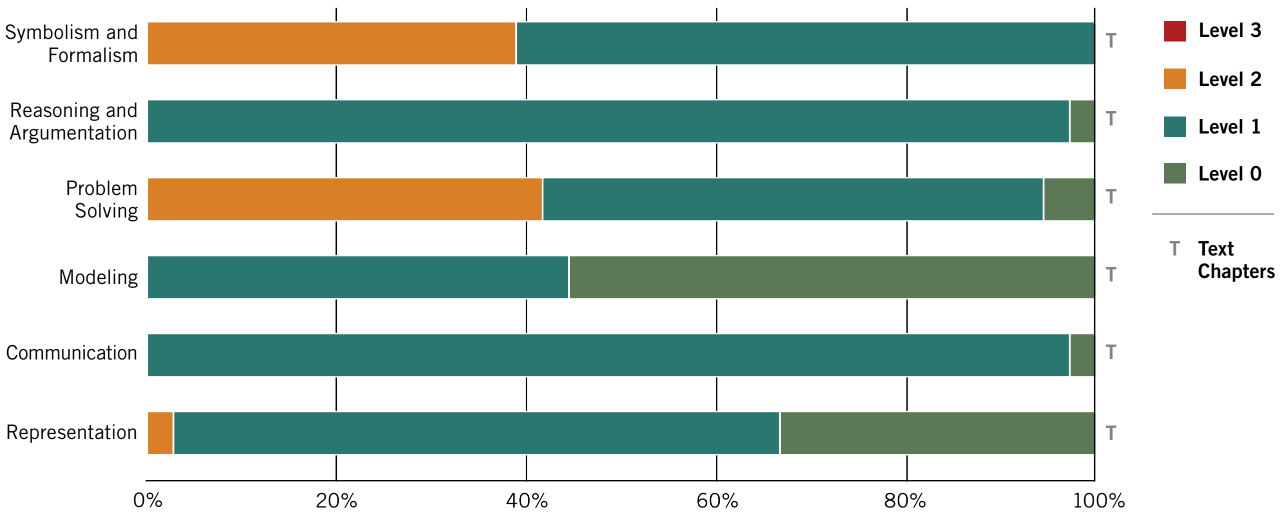
101 Course Title	Initial Required Mathematics Course
Nursing Practice I	Applied Mathematics or College Algebra
Nursing I	Applied College Math
Fundamentals of Nursing	Math of Medical Dosages
Anatomy and Physiology	No requirement
Introduction to Nursing Practice	No requirement
Nursing Theory and Science 1	Intermediate Algebra

CHART F1 *Average Percent of Text Chapters Containing CCSSM Domains for Biotechnology Mathematics Courses*



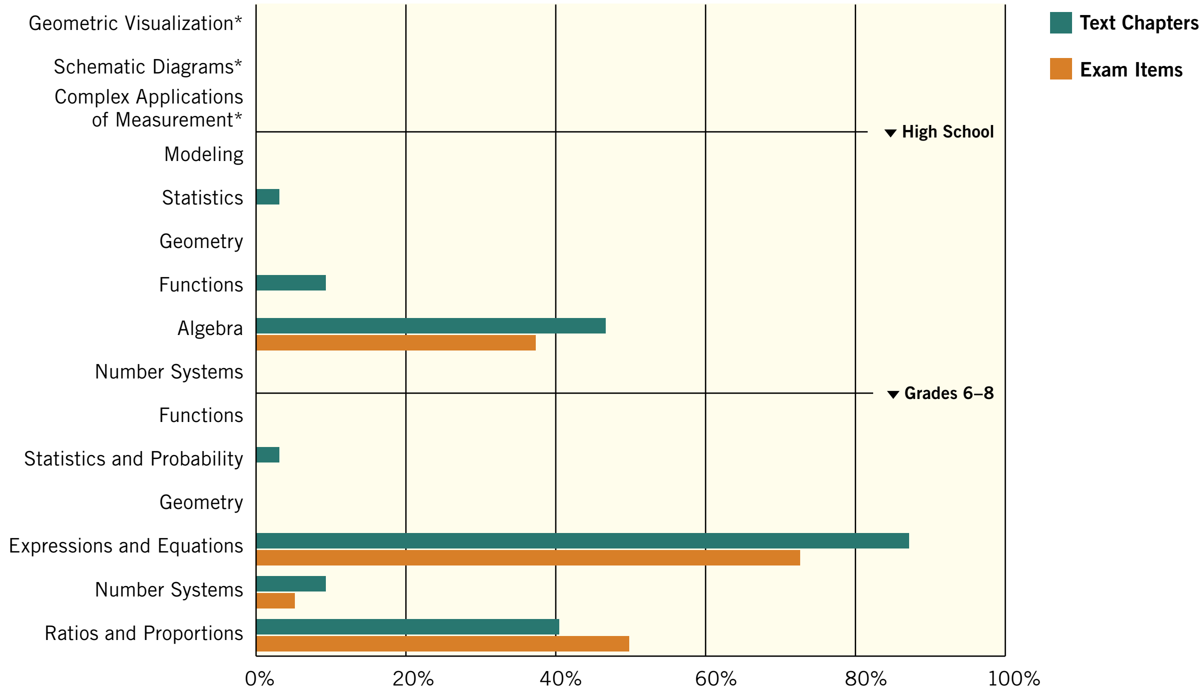
Non-CCSSM domains added by the panel.

CHART F2 *Average Percent of Text Chapters Rated at Each PISA Level for Biotechnology Mathematics Courses*



No exams were included in the materials supplied by the Biotechnology Mathematics Courses.

CHART F3 *Average Percent of Text Chapters and Exam Items Containing CCSSM Domains for Business Mathematics Courses*



**Non-CCSSM domains added by the panel.*

CHART F4 *Average Percent of Text Chapters and Exam Items Rated at Each PISA Level for Business Mathematics Courses*

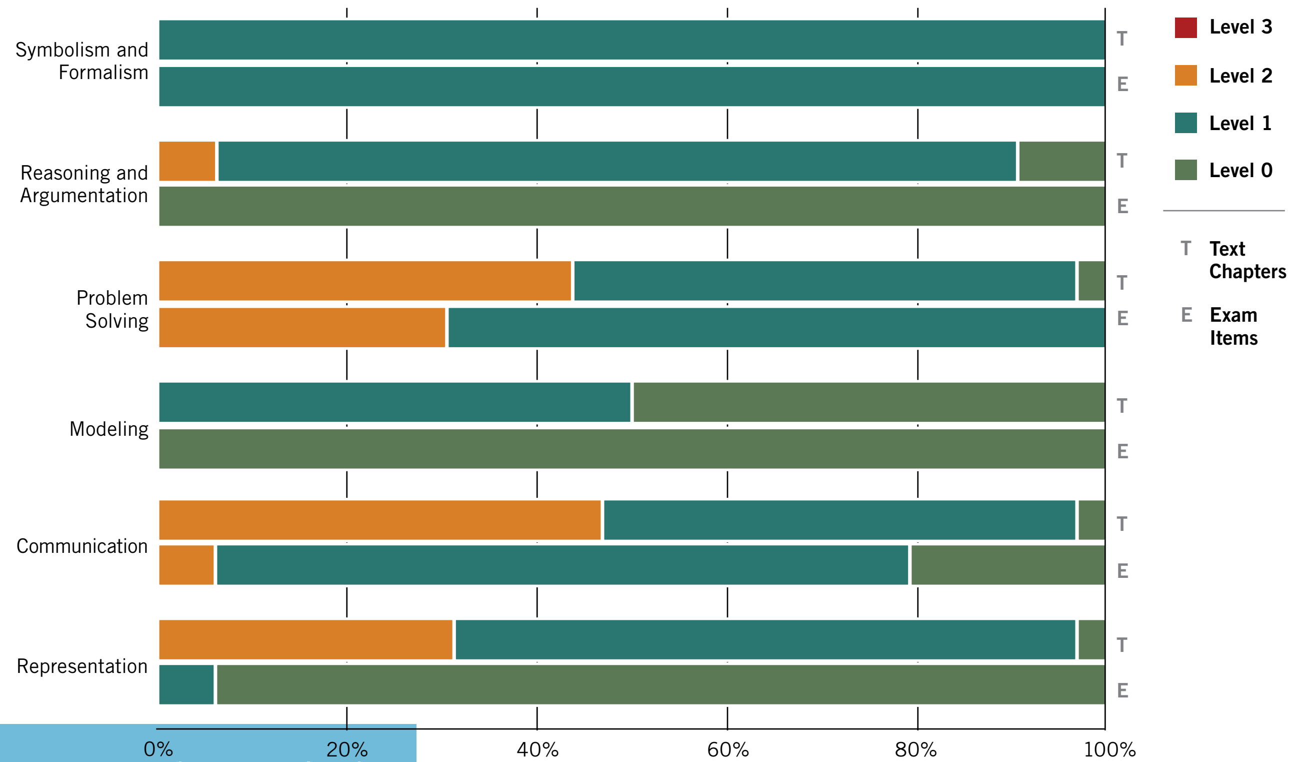
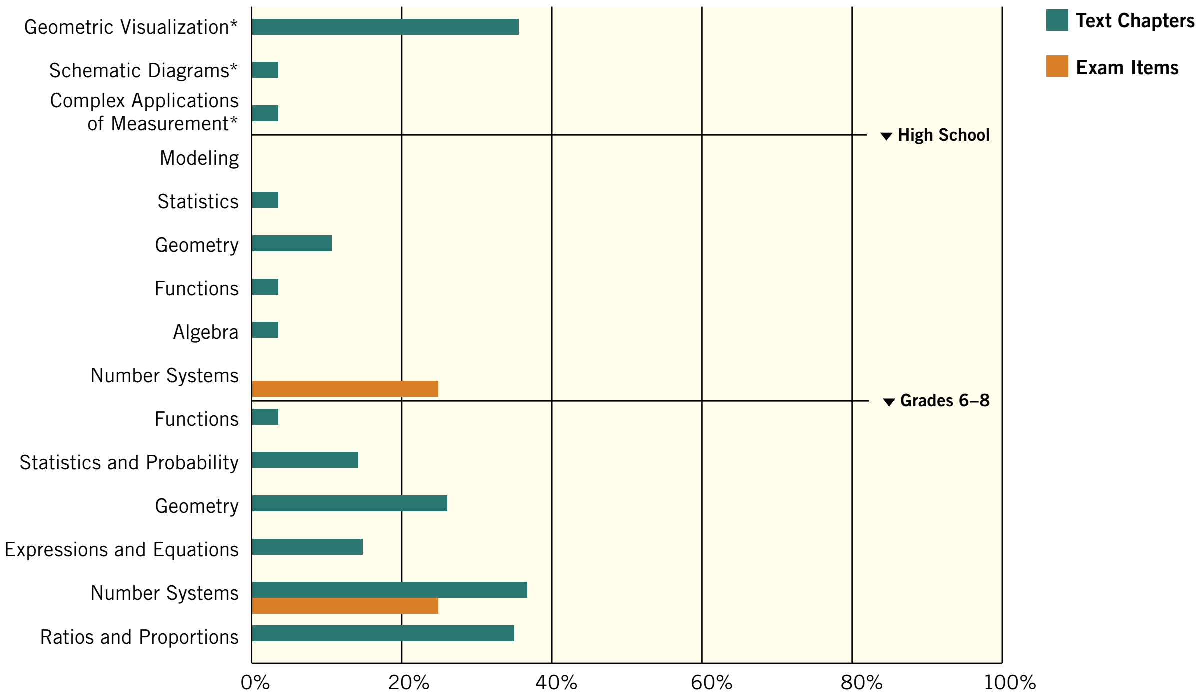


CHART F5 *Average Percent of Text Chapters and Exam Items Containing CCSSM Domains for Early Childhood Education Mathematics Courses*



*Non-CCSSM domains added by the panel.

CHART F6 *Average Percent of Text Chapters and Exam Items Rated at Each PISA Level for Early Childhood Education Mathematics Courses*

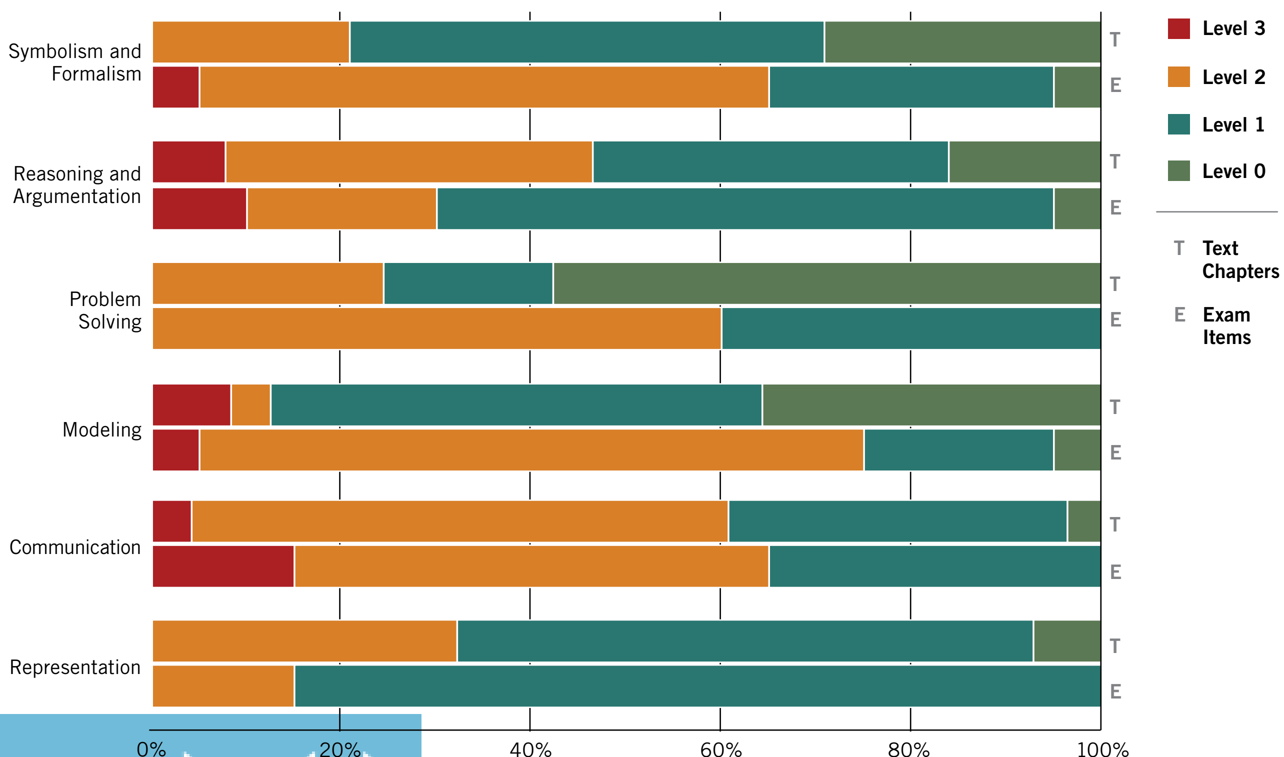
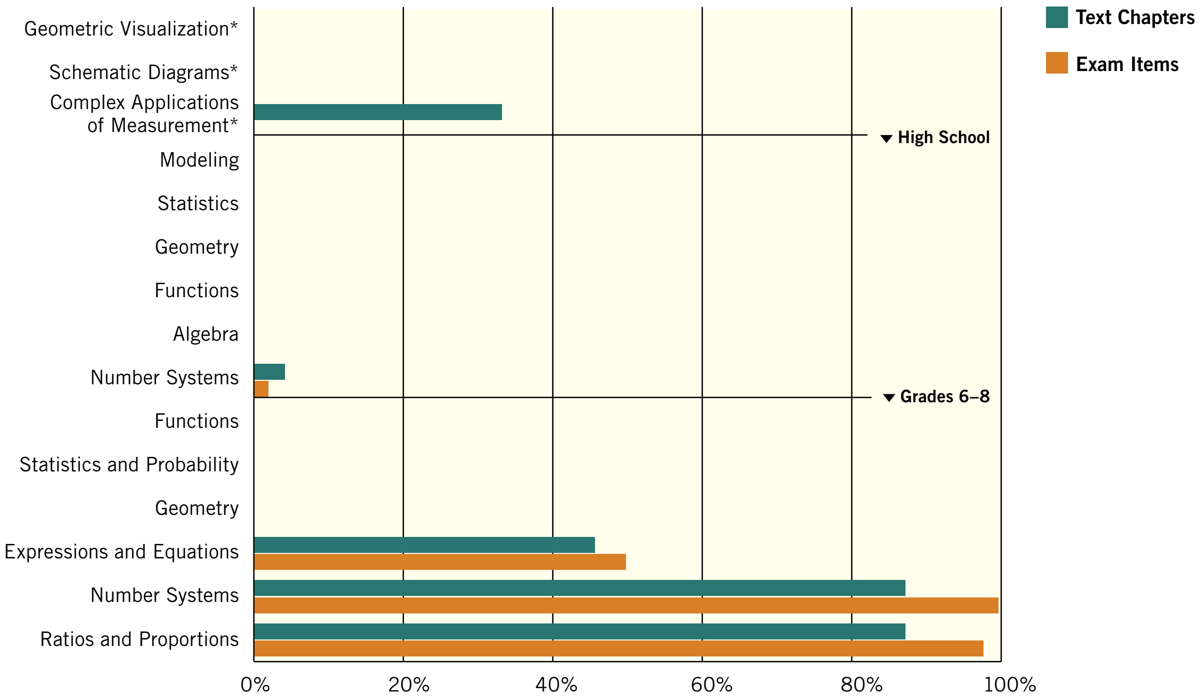


CHART F7 *Average Percent of Text Chapters and Exam Items Containing CCSSM Domains for Nursing Mathematics Courses*



*Non-CCSSM domains added by the panel.

CHART F8 *Average Percent of Text Chapters and Exam Items Rated at Each PISA Level for Nursing Mathematics Courses*

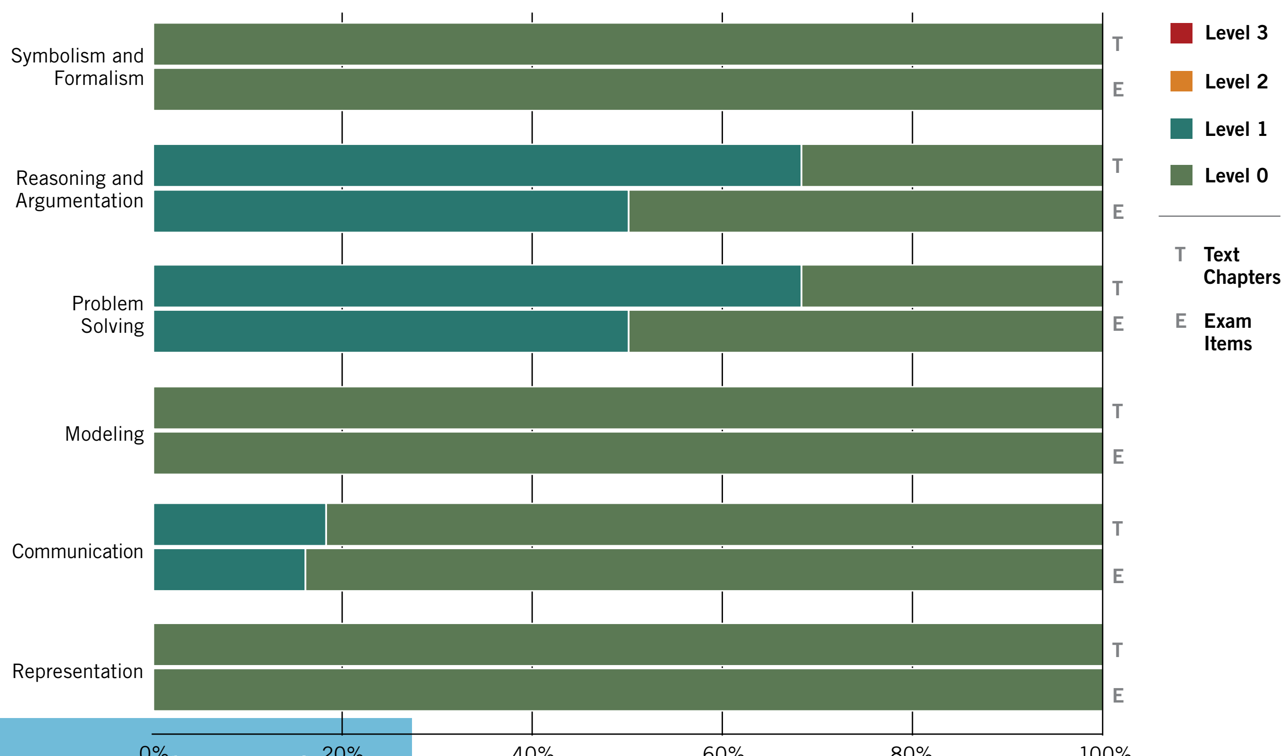


CHART G1 *Prerequisites Required for College Algebra Courses: Based on the Percent of Text Chapters and Exam Items that Utilize Each CCSSM Domain*

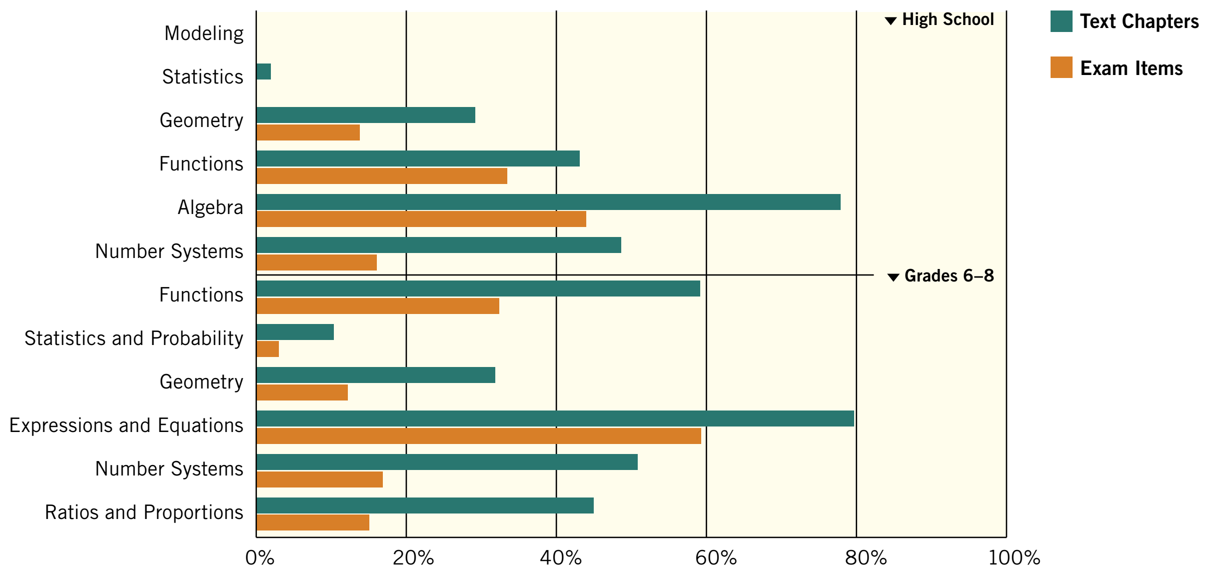


CHART G2 *Prerequisites Required for General Mathematics Courses: Based on the Percent of Text Chapters and Exam Items that Utilize Each CCSSM Domain*

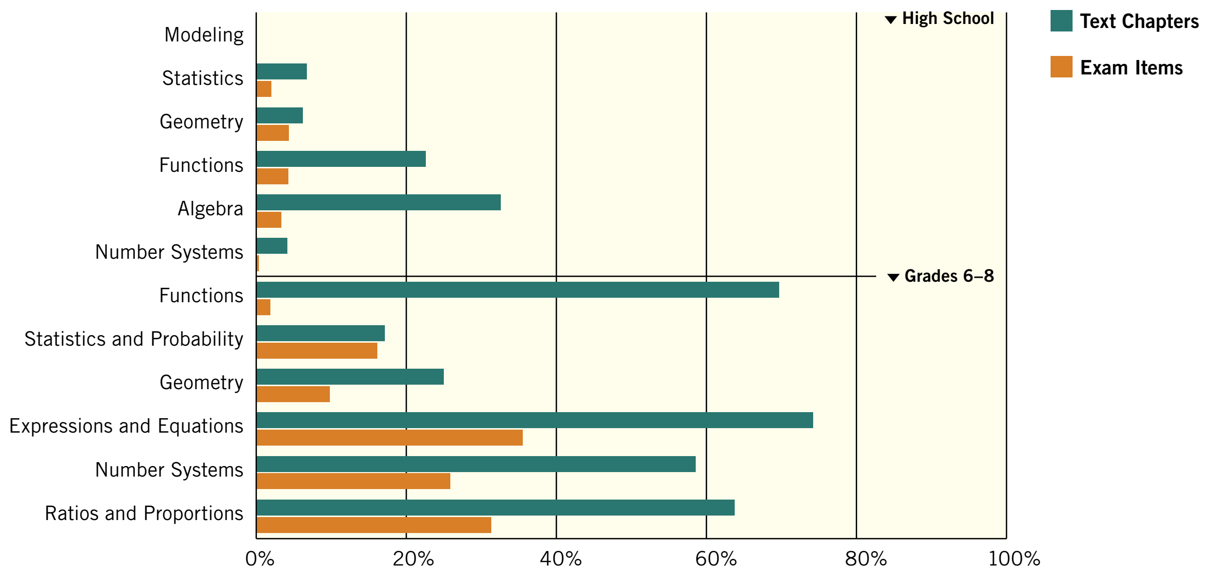


CHART G3 *Prerequisites Required for Statistics Courses: Based on the Percent of Text Chapters and Exam Items that Utilize Each CCSSM Domain*

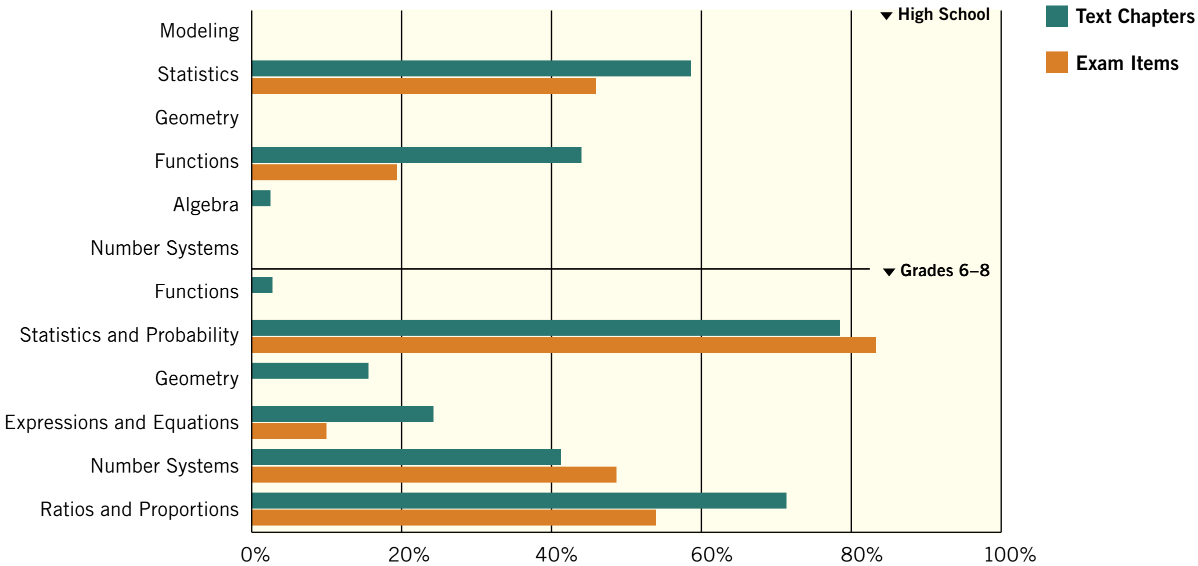


CHART G4 *Prerequisites Required for Biotechnology Mathematics Courses: Based on the Percent of Text Chapters that Utilize Each CCSSM Domain (No exams were included in materials received for these courses.)*

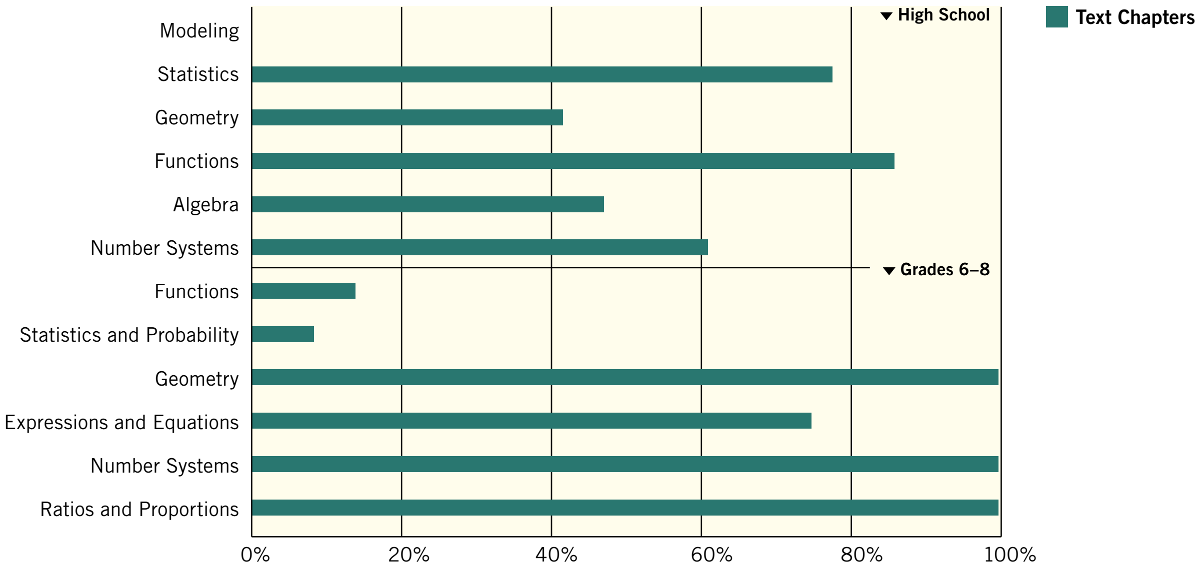


CHART G5 *Prerequisites Required for Business Mathematics Courses: Based on the Percent of Text Chapters and Exam Items that Utilize Each CCSSM Domain*

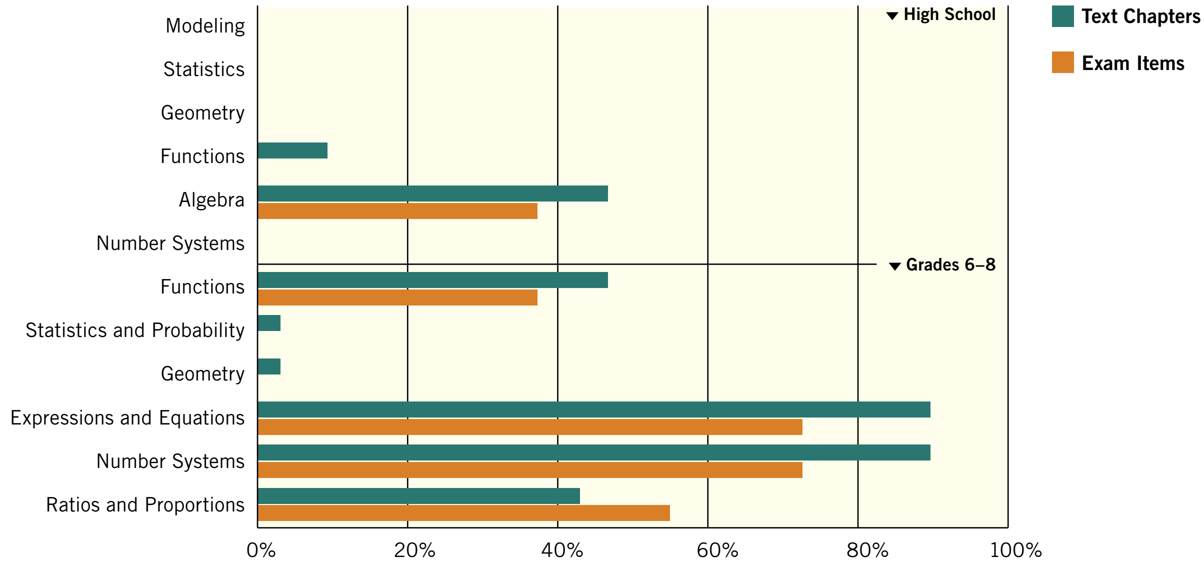


CHART G6 *Prerequisites Required for Early Childhood Education Mathematics Courses: Based on the Percent of Text Chapters and Exam Items that Utilize Each CCSSM Domain*

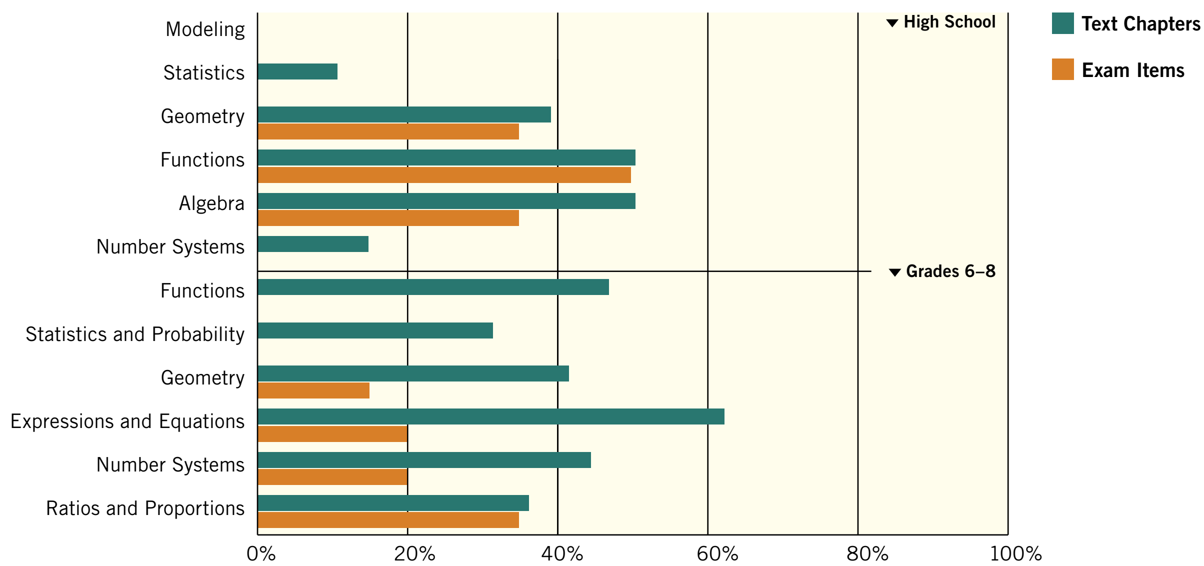
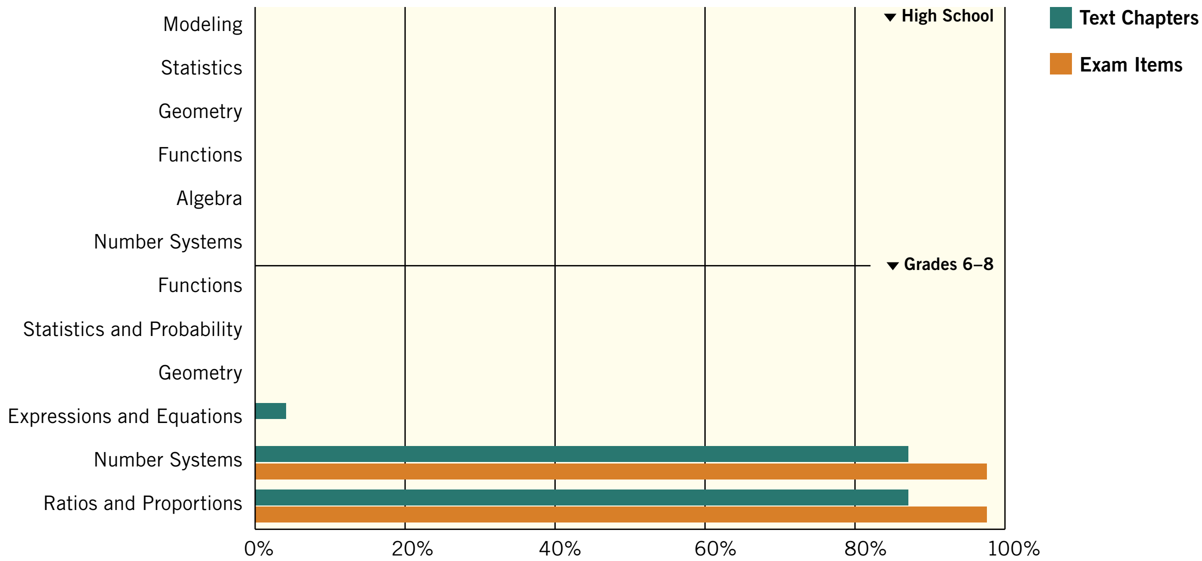


CHART G7 *Prerequisites Required for Nursing Mathematics Courses: Based on the Percent of Text Chapters and Exam Items that Utilize Each CCSSM Domain*



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